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Dr. Rick Jostes asked me to give a talk for a National Research Council committee that is investigating aspects of a phased-array radar named PAVE PAWS. Dr. Jostes suggested that I discuss the relation of statements by Prof. Kurt Oughstun to my recent work on exponential decay of radiation in lossy materials. With that incentive, I found that most of the 38-years' of precursor literature is irrelevant to PAVE PAWS. Indeed, I will first list incident pulses mentioned by this literature, but whose spectra are *not* separated from DC. (DC is a synonym for frequency=0.) Such pulses cannot be produced by PAVE PAWS, which broadcasts from 420–450 MHz or, equivalently, 435 MHz ±3.5%. I will also list parts of the literature that regard these types of pulses that PAVE PAWS cannot produce. This executive summary will also sketch answers to questions asked during my talk. This report's section after the transparencies will substantiate the executive summary.

In conclusion, an entire body of scientific literature that goes back 88 years is irrelevant to PAVE PAWS. This may simplify the committee's deliberations on PAVE PAWS.

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5 March 2003

Good Morning

This paper was submitted to DTIC last Fall for documentation. We have been unable to find it in the system so we are resubmitting it.

Thank you for your assistance in this matter. If you have any questions please call me at DSN 478-2059 or commercial 781-377-2059.

Sheila C. Belliveau

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PAVE PAWS Radiation
Decays Exponentially in Lossy Materials

Enlarging on a Talk for
National Research Council
Project BRER-K-01-01-A
Woods Hole, Mass.
Sept. 9, 2002

by
Tom Roberts
Air Force Research Laboratory
Hanscom Air Force Base, Mass.

TABLE OF CONTENTS

EXECUTIVE SUMMARY

Dr. Rick Jostes asked me to give a talk for a National Research Council committee that is investigating aspects of a phased-array radar named PAVE PAWS. Dr. Jostes suggested that I discuss the relation of statements by Prof. Kurt Oughstun to my recent work on exponential decay of radiation in lossy materials. With that incentive, I found that most of the 88-years' of precursor literature is irrelevant to PAVE PAWS. Indeed, I will first list incident pulses mentioned by this literature, but whose spectra are *not* separated from DC. (DC is a synonym for frequency=0.) Such pulses cannot be produced by PAVE PAWS, which broadcasts from 420–450 MHz or, equivalently, 435 MHz ±3.5%. I will also list parts of the literature that regard these types of pulses that PAVE PAWS cannot produce. This executive summary will also sketch answers to questions asked during my talk. This report's section after the transparencies will substantiate the executive summary.

In conclusion, an entire body of scientific literature that goes back 88 years is irrelevant to PAVE PAWS. This may simplify the committee's deliberations on PAVE PAWS.

The following time-dependent pulses have DC or near-DC content. These pulses cannot be produced by PAVE PAWS.

(1) step-modulated sine pulses
(2) step-modulated cosine pulses
(3) finite duration, integer-cycle sine pulses
(4) finite duration, non-integer-cycle sine pulses
(5) gaussian-modulated sine pulses
(6) gaussian-modulated cosine pulses
(7) hyperbolic-tangent-modulated sine pulses
(8) hyperbolic-tangent-modulated cosine pulses
(9) the Dirac δ-function pulse
(10) any pulse whose Fourier transform $q(\omega)$ satisfies the Olver-method condition (a) on p. 159 of Oughstun and Sherman's book
(11) pulses whose spectra are not separated from DC
(12) pulses that decay slower than exponentially in at least one lossy material

---

The following work is largely or entirely devoted to pulses that have DC or near-DC content. Such work is largely or entirely irrelevant to PAVE PAWS.

[1] Most of the work by L Brillouin and A Sommerfeld on Lorentz models, published in 1914 and 1932 and translated into English in L Brillouin, *Wave Propagation and Group Velocity* (Academic, 1960).

[2] Lorentz-model asymptotics in Sect. 7.11(g) on pp. 321–324 of JD Jackson, *Classical Electrodynamics* 2nd edn. (Wiley, 1975).

[3] Decay rates derived from ibid. and appearing in endnote 30 of TM Roberts and PG Petropoulos, "Asymptotics and energy estimates for electromagnetic pulses in dispersive media," J. Opt. Soc. Am. A **13**, 1204–1217 (1996). Also, ——, "Asymptotics and energy estimates for electromagnetic pulses in dispersive media: addendum," J. Opt. Soc. Am. A **16**, 2799–2800 (1999).

[4] Debye-model (Sec. 2.2) and Lorentz-model (Sec. 2.3) asymptotics in M Kelbert and I Sazonov, *Pulses and Other Wave Processes in Fluids* (Kluwer, 1996).

[5] Most asymptotics published by KE Oughstun *et al.*, including [A] KE Oughstun, "Propagation of optical pulses in dispersive media," Ph.D. dissertation (University of Rochester, 1978; published by UMI, Ann Arbor, Mich.), [B] KE Oughstun and GC Sherman, *Electromagnetic Pulse Propagation in Causal Dielectrics* (Springer-Verlag, 1994), and [C] most other published work by KE Oughstun *et al.*

[6] Most of the work on Lorentz or Debye models published by RA Albanese *et al.*, including "Short-rise-time microwave pulse propagation through dispersive biological media," J. Opt. Soc. Am. A **6**, 1441–1446 (1996). This paper is in the journal issue's special feature, "Mathematics and Modeling in Modern Optics," KE Oughstun and JJ Stamnes eds.

The section of this report after the transparencies will substantiate the itemized statements above. The "Answers to Questions asked During the Talk" section will also respond more fully than I did during my talk: The "Answers..." offer an intuitive connection between algebraic decay and the zeros of frequency-dependent coefficients of exponential decay. The Answers also show that a new model mentioned by Prof. Oughstun's talk would often yield algebraic decay in circumstances for which exponential decay was newly claimed by Prof. Oughstun in that talk. The two previous sentences respond to questions from a committee member. The Answers include my vita, which an audience member wanted to see and which committee members discussed publicly without revealing detail.

Preface

I rehearsed my presentation into a tape recorder the night before my talk. I did this because I planned to give a shorter version of the talk in a different setting perhaps a month or so later. It would be easier then, I thought, to find the right words on tape than to find them in reaction to reading my transparencies.

Following each transparency photocopy is a narrative drawn largely from my 9/8/02 recording. Supplementary information follows some narratives. Attendees who understood my talk may want to go directly to the Justification and Answers sections, near the end of this report.

PAVE PAWS RADIATION DECAYS EXPONENTIALLY IN LOSSY MATERIALS

For

National Research Council
Project BRER·K·01·01·A
Woods Hole, MA
Sept. 9, 2002

By

Tom Roberts
Air Force Research Lab
Hanscom AFB, MA

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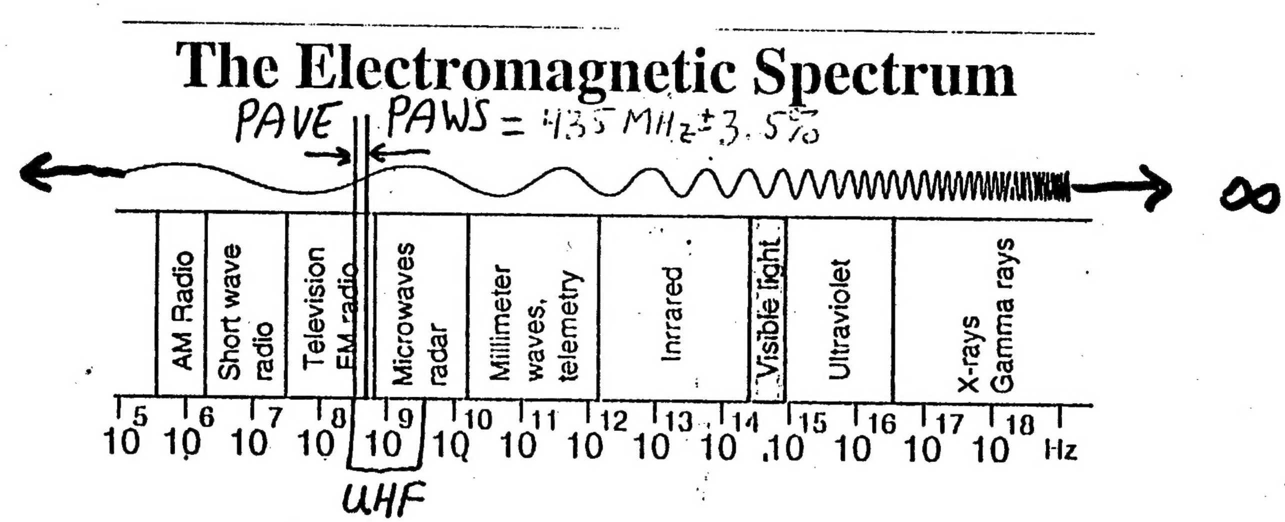


Transparencies, Narratives, and Supplements
First Transparency
PAVE PAWS Radiation Decays Exponentially in Lossy Materials

Narrative

Good afternoon. I am pleased to be here.

I'll talk today about the type of pulses that PAVE PAWS can produce. I'll also talk about a type of pulses that PAVE PAWS cannot produce, but that has entered nevertheless into discussions about PAVE PAWS. And I'll also discuss the ways in which both of these types of pulses propagate through lossy materials such as human tissues, and through lossy models such as the Lorentz model. That covers the whole talk.

I won't talk about health or safety because I don't know much about those subjects. I'll talk about only electromagnetics.

At the bottom of the transparency I've sketched the spectrum of the pulses that PAVE PAWS broadcasts. This spectrum is 420–450 MHz. This is not first-hand experience for me. It's what I've read in Air Force technical reports. My most recent source is the preliminary-measurement report on the www.pavepaws.org web site. The earliest report that I know of that mentions the 420–450 MHz band for PAVE PAWS is the 1980 environmental impact statement for a PAVE PAWS site in Calif. I'm now assuming, for the reasons just mentioned, that the spectrum of PAVE PAWS radiation is 420–450 MHz. For the purpose of my talk, all that matters is that the spectrum is hundreds of MHz removed from DC. DC is a synonym for frequency=0. This band is also narrow in the sense that the band is the center frequency ±3.5%: It's 435 MHz ±3.5%. The spectrum restricts the type of pulses that can be broadcast. And this ±3.5% is the tightest restriction I know of on the pulses shapes that PAVE PAWS can produce. This seems to be the definitive restriction.

Supplement

The 1980 PAVE PAWS report is: Department of the Air Force, "Operation of the PAVE PAWS radar system at Beale Air Force Base, California. Part 1. Basic EIS & Appendices" (July 1980) technical report AFSC-TR-80-09, also known as ADA 088 320. "EIS" (above) means environmental impact statement. Markings state that the report has been approved for public release, with unlimited distribution. The 420–450 MHz band is mentioned in the next-to-last paragraph of p. 1-5, which is the 5th page of Chapt. 1.

ALGEBRAIC AND EXPONENTIAL DECAY

"In a phased array system like PAVE PAWS,...[in] the side lobes.... [the] most important effect is that the radiation no longer decays exponentially in lossy materials" [but it decays more slowly]. — K. Oughstun, Microw. News, Mar./Apr. 2002

	Examples (x = depth)			
Algebraic Decay	$x^{-1/3}$,	$x^{-1/2}$,	$x^{-2/3}$;	x^{-a} (slower)
Exponential Decay		10^{-x}	;	10^{-bx} (faster)

"Radiated pulses decay exponentially in lossy materials [far from] antennas" — Me, Electronics Letters, 7/4/02

submitted 2/28/02
peer reviewed
published w/o revision

Second Transparency

Algebraic and Exponential Decay

Narrative

When I was first asked whether I was willing to talk today, it was explained to me that the committee often finds it useful to have presentations that balance each other. It was suggested that my recent work on exponential decay and the things that Prof. Oughstun may say about algebraic decay could balance each other.

So here and on the next transparency I will discuss a statement that he made [top] with a statement that I made [bottom]. The next transparency will explain the research behind my statement and research that may be behind Prof. Oughstun's statement.

I'll start with Prof. Oughstun's statement. It is from an interview of him by *Microwave News*, which appeared in that newsletter's March/April issue this year. Prof. Oughstun said many things in that interview. I have excerpted parts of three sentences, in the order in which those sentences were printed; I put them together with ellipses; and I changed the wording to make the composite sentence flow. The result is a sentence that Prof. Oughstun never really said. But the sentence does use his words, and I think it represents reasonably well the gist of what he did say at one point. He said, "In a phased array system like PAVE PAWS, ... [in] the side lobes... [the] most important effect" — and here, in context, he meant the electromagnetic effect that he thinks is potentially the most important for biological systems — "The most important effect is that the radiation no longer decays exponentially in lossy materials..." He made it clear later in the interview that he thinks the decay is slower than exponential.

On the other hand, I have an article in *Electronics Letters*, which is a peer-reviewed journal. My article was in the July 4 issue this year. The title of that paper is "Radiated pulses decay exponentially in lossy materials [far from] antennas."

You can see right away that these two statements overlap. For example, they both mention "lossy materials." And soon I'll mention other topics that the statements have in common. But for now I want to note that each statement was made without knowledge of the other statement. When I wrote my paper in February of this year I could not have known that the next month's newsletter, which I would not get a copy of until May, would have this statement by Prof. Oughstun. And when Prof. Oughstun gave that interview, he could not have known of my statement that would be published months later.

The two independent statements do have "lossy materials" in common. And there's more that they have in common.

Here, in the title of my paper, there's "lossy materials [far from] antennas." You'll see what I mean by "[far from] antennas" if you read my paper's introduction and the next few sentences after that. It won't take long to read the introduction because my paper is only a page and a half long, which is this jour-

nal's maximum length for papers. And in that introduction you'll see that "[far from] antennas" means only that the pulses that are incident on the materials would have spectra that are separated from DC. That is, "[far from] antennas" means, in context, "separated from DC." As you know, PAVE PAWS pulses are separated from DC by hundreds of MHz. So my statement includes PAVE PAWS pulses. Of course I could not have known when I wrote this paper title in February that exponential decay would later become an issue for PAVE PAWS. The title was meant only to refer to separated-from-DC pulses produced by any antenna. But my title statement does apply to PAVE PAWS.

Prof. Oughstun's statement mentions PAVE PAWS. So he and my statements have both lossy materials and PAVE PAWS in common.

The two statements also mention exponential decay. My statement says that the pulses decay exponentially; Prof. Oughstun's statement says they decay slower than exponentially. But exponential decay and slower-than-exponential decay are mutually exclusive concepts. Therefore, Prof. Oughstun and my statements cannot both be correct. At least one must be incorrect.

The tipping point of these two statements is the concept of exponential decay, which I will now describe. Exponential decay is simply a power-of-ten decay. In the course of this talk you will see three ways of measuring the size of a pulse. These sizes are the peak value, the peak power, and the energy of the pulse. If one of these sizes, say the peak value, decays as a power of ten — that is, if the peak value decays as ten to the power of a negative constant times the depth x as the pulse propagates through a material — then the peak value is said to decay exponentially. And if the energy decays as a power of ten, then the energy is said to decay exponentially.

Prof. Oughstun's statement does not specify the rate of slower-than-exponential decay that he thinks will occur in these circumstances. Although Prof. Oughstun's statement does not actually mention algebraic decay, it is at least informed by his 25-years' experience in predicting algebraic decay of pulses in Lorentz models. I will now describe algebraic decay.

Algebraic decay is a depth-to-a-negative-constant decay. I've given three examples [$x^{-1/3}$, $x^{-1/2}$, $x^{-2/3}$] that are taken from peer-reviewed publications by various people — not just by Prof. Oughstun — involving predictions of algebraic decay for Lorentz models. Algebraic decay turns out to be usually slower than exponential. There are some exceptions, but algebraic decay is still *typically* slower than exponential decay. At large-enough depths, however, algebraic decay is always slower than exponential decay. There is no exception as depth goes to ∞.

Although algebraic decay is typically slower than exponential decay, it's still not clear whether Prof. Oughstun meant to refer to algebraic decay when he talked of slower-than-exponential decay.

Next I'll describe the research history that may support his statement.

Then I'll tell you about the research that does support my statement.

Supplement

In this talk, the peak value of a time-dependent pulse $p(t)$ is the largest value of $|p(t)|$. By "peak power" I mean the peak power density, which is proportional to the square of the peak value. By "energy" I mean the energy density. These concepts are explained in standard electromagnetics texts for undergraduates in physics and electrical engineering.

Readers may be puzzled that my narrative does not resolve its stated uncertainty regarding the intent of Prof. Oughstun's statement quoted by this transparency. After all, Prof. Oughstun gave the talk immediately before mine. But he did not respond to what I said. Indeed, I did not see Prof. Oughstun in the room during my talk. Three attendees later said that he left the room as soon as I began talking. One of these attendees added that Prof. Oughstun would occasionally return within normal hearing range of me but not within view, and that he always stayed only briefly before walking away. I mention this only to explain why my narratives do not recount statements made by Prof. Oughstun during my talk. Specifically, no such statement exists.

Decay of Brillouin Precursors in Lorentz Models

Authorship	Inc. Pulses (O&S) $\Rightarrow$ Decay
Many people. Among them: **Oughstun & Sherman 1994** pp. 159, 275–280, 351–352	Fourier transform analytic in domain that incl. DC $\Rightarrow$ Algebraic
Me, July 4, 2002 Short derivation accessible to many undergrads. Also applies to all lossy materials. 8 numerical examples	Spectrum separated from DC $\Rightarrow$ Exponential

$\therefore$ Slower-than-exp. decaying pulses have DC or near-DC content.

Third Transparency

Decay of Brillouin Precursors in Lorentz Models

Narrative

This transparency compares 88-years' of published research with my page-and-a-half paper from July 4.

All of that 88-years' of work regards the Lorentz model. The Lorentz model was developed by a German named Drude in his course notes, which led to a 1894 textbook titled *Physics of the Aether.* The model was meant to account for phenomena observed near the absorption bands of the spectra of visible light that had been transmitted through a stellar plasma, Earth's ionosphere, or a thin film of Iodine or other material deposited on a glass plate. This model has for various reasons become popular among theorists. But if someone uses a Lorentz model to represent a particular material, then it is fair to ask for evidence that the model fits.

Many people have studied how pulses decay in Lorentz models. This was done first by Brillouin and Sommerfeld in 1914. They studied how integer-cycle sinusoid pulses would decay in a Lorentz model. An integer-cycle-sine pulse is a concept you will hear repeatedly in this talk. It is a pulse that is 0 for awhile, then it oscillates sinusoidally for an integer number of periods, and then it remains 0. Brillouin predicted that this type of incident pulse would have a specified rate of algebraic decay as it went through a Lorentz model. Prof. Oughstun, with many more co-authors than are mentioned here, also predicted specified rates of algebraic decay for integer-cycle sines and for other incident pulses. I too have made similar predictions, as have others whose work has nothing to do with Prof. Oughstun or me. As far as I know, from 1914 through July 3, 2002, every time anyone studied the rate of decay of a pulse in any type of dispersive model they predicted algebraic decay of one rate or another. That's to my best knowledge. I haven't read all the literature, but I've read a lot.

Prof. Oughstun has a published statement that, to my knowledge, makes the world's broadest claim that a large class of pulses decays algebraically. This is from the proceedings of an IEEE meeting in the year 2000. Prof. Oughstun wrote there that all ultra-wideband pulses will decay in Lorentz models at an algebraic rate that he specified. But that statement is not accompanied by a derivation, nor is it accompanied by a citation of a derivation that would show that the statement is correct. His statement from 2000 is therefore a conjecture. It is not clearly labeled a conjecture, but it is indeed a conjecture. I mention it here only because I'm reviewing the literature. I will not mention Prof. Oughstun's conjecture again.

The next-most-broad statement that many pulses decay algebraically is in Oughstun and Sherman's book from 1994. The cited pages of the book show that they predict that at least 4 types of incident pulses will decay at an algebraic rate that they specify. The cited pages also show that all these example pulses had this specific mathematical property ["analytic in a domain..."]. It takes specialized

knowledge to understand this property. It is a technical math property that may be unfamiliar to almost everyone in this room. It is, however, in part of the standard undergraduate curriculum for math majors. The relevant course is usually called complex variables or complex analysis. The stated math property is very specific here, it's even more specific in Oughstun and Sherman's original, and it makes perfect sense. It's a fine statement. And for several types of pulses that have this math property, they do derive algebraic decay. That's an essential point: Namely, the broadest, derived statement of algebraic decay is for a class of pulses that has this specific math property. I will return to this essential point soon.

My work from July, on the other hand, assumes only that the incident spectrum is separated from DC. PAVE PAWS is an example. Using only the separation from DC, I showed that the decay would be exponential.

Please notice that exponential decay and algebraic decay are mutually exclusive behaviors. Assuming that Oughstun and Sherman's derivation is correct, and assuming that my derivation is correct, it follows that these two sets of incident pulses are mutually exclusive. Indeed, anyone who knows complex analysis can recognize almost immediately that there is no pulse that has both of these properties. ["Both" means the analyticity and the separated-from-DC properties.] To confirm that this is immediately recognizable, I took two undergraduate texts out of my bookcases at work and I found the basis for this conclusion in each book. In each undergraduate textbook, this was in the homework problems.

It follows that the pulses studied in Oughstun and Sherman's book all have DC or near-DC content, which cannot be produced by PAVE PAWS. Prof. Oughstun, in his talk immediately preceding mine, explained how almost all of his work uses the Olver method. Indeed, the analyticity condition mentioned here is item (a) in the book's description on p. 159 of the Olver method. So every time that the Olver method is used as Oughstun and Sherman's book uses it, the results are irrelevant to PAVE PAWS for spectral reasons. Indeed, this same Olver-method condition (a) appears in Prof. Oughstun's Ph.D. dissertation from 1978. Having read almost all of Prof. Oughstun's work, I assure you that almost all of his asymptotics from 1978 to present use that Olver-method condition. Almost all of his asymptotics from 1978 to present, therefore, are irrelevant to PAVE PAWS for the spectral reasons mentioned earlier.

There are, of course, other people who studied algebraic decay without reference to the Olver method. I am one of those people. There are others. But my July result shows that all those algebraic decay results must have assumed, at least implicitly, that the incident-pulse spectrum was *not* separated from DC.

These circumstances explain how it can be that various people studied pulse decay in Lorentz models across 88 years and always got algebraic decay. It started with Brillouin and his integer-cycle sine in 1914, and people just followed his lead using this and other types of pulses that happened to have DC or near-DC content. And it also happens that none of this body of literature is relevant to

PAVE PAWS.

As far as I know, my July 4 paper is the only one to have ever studied pulse propagation for Lorentz models and to have concluded that the decay could be exponential. Every pulse that PAVE PAWS produces happens to be separated from DC, and in every case the decay is exponential in all lossy materials. The derivation of this is sketched in my page-and-a-half paper near the end of my handout for the committee. The full, line-by-line derivation follows my paper in the handout. It's a 5-line calculus derivation. [The paper and derivations are also in the last pages of this report.]

My previous transparency showed Prof. Oughstun's interview statement that some PAVE PAWS pulses decay slower than exponentially in lossy materials. I've just shown that his statement is incorrect. I don't know where his statement came from. It may have come from the 88-year history of research into pulses that happen to be not producible by PAVE PAWS. But I don't really know why he said what he did say.

I would like to spend a little time describing my paper from July 4. After all, it's the reason I was asked to talk here. My paper is mentioned here on this transparency, whose title mentions Lorentz models. The paper does apply to Lorentz models, but it applies to all other lossy models and to all lossy materials. Human tissues are lossy, hence the relevance to this committee. The derivations in my paper are also short. This was necessary because the journal's length limit was a page and a half. The committee, however, has my full line-by-line derivations. There are two derivations, each composed of 5 steps of calculus. Many people who use a lot of integral calculus and who are familiar with the complex-valued exponential function can follow the first 5-step derivation. The second 5-step derivation is a little more advanced, but many undergraduates could still follow it, I think. I would be happy if someone would go through my short derivations, or get someone else to go through them, and decide on that basis whether I am right or wrong. The two short derivations are, in any case, a brief part of my page-and-a-half paper. I also crammed 8 numerical examples into that page and a half. It's not just theory: there's numerics, too.

Supplement

This transparency obtains the main results of my talk. I have several supplementary statements.

First, I cite P Drude, *Physik des Aethers*, in German, (Ferdinand Enke, Stuttgart, 1894) Sec. 10 of Chapt. 10.

Second, at least two committee members asked questions about a new, unpublished result that Prof. Oughstun mentioned in his talk immediately preceding mine. I was unprepared to answer. A full answer follows in the Answer section near the end of this report. Prof. Oughstun's new, unpublished result is evidently incorrect.

Third, the paragraph about ultra-wideband (UWB) pulses is from my 9/8/02 rehearsal. I omitted the paragraph from my talk to compensate for time taken to answer committee questions. The claim about UWB pulses is from KE Oughstun and PD Smith, "On the accuracy of asymptotic approximations in ultrawideband signal, ultrashort-pulse, time-domain electromagnetics," in *Digest of the 2000 IEEE Antennas and Propagation Society International Symposium* Vol. 2 (IEEE, Piscataway, New Jersey, 2000) pp. 685–688. The publication's first sentence and first equation are evidently the only assumptions regarding the incident pulse: Namely, it's a UWB pulse that has a Fourier transform. Without further restriction on the pulse, algebraic decay is claimed 3 lines above the figures of p. 687. I cannot find a justification for this published claim that all UWB pulses that have Fourier transforms will decay algebraically in both the Lorentz and Debye models.

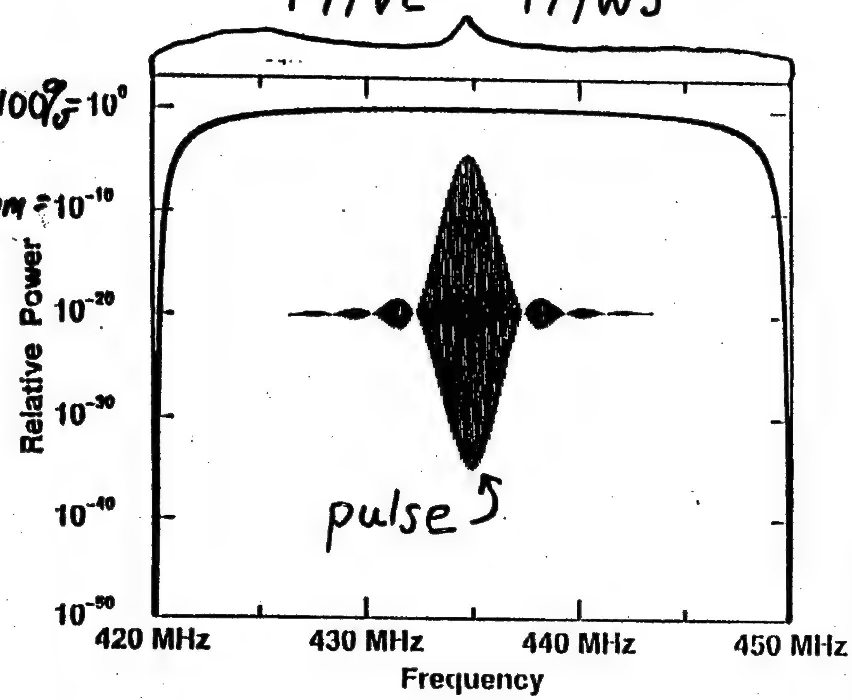
Fourth, to show that many math majors could immediately confirm that the analyticity and separated-from-DC properties are mutually exclusive, I looked in two books. One is JD Paliouras, *Complex Variables for Scientists and Engineers* (Macmillan, 1975) pp. 290 and 293. Exercise 28.15 there requires the student to prove Theorem 7.2, whose corollary ("REMARK 2" on p. 293) immediately implies that *only* the zero pulse [$p(t) = 0$ for all time] has both the analyticity and the separated-from-DC properties. Paliouras' textbook is meant for undergraduates. This material is also covered in RV Churchill and JW Brown, *Complex Variables and Applications* 5th edn. (McGraw-Hill, 1990) exercise 8 on p. 181. Exercise 8 includes a hint. This textbook is meant for "seniors and graduate students majoring in mathematics, engineering, or one of the physical sciences." Although both textbooks are intended for science and engineering majors, as well as for math majors, the course is usually offered by the math department. To be complete, Oughstun and Sherman's book's use of the Olver method implicitly assumes that the incident pulses are not precisely 0 in their domains of analyticity. It follows from undergraduate homework that Oughstun and Sherman's pulses are not separated from DC.

Fifth, Prof. Oughstun did not respond to my presentation of this transparency. Relevant circumstances are mentioned at the end of the supplementary information for my talk's second transparency.

Inside PAVE PAWS Band

Narrow Band

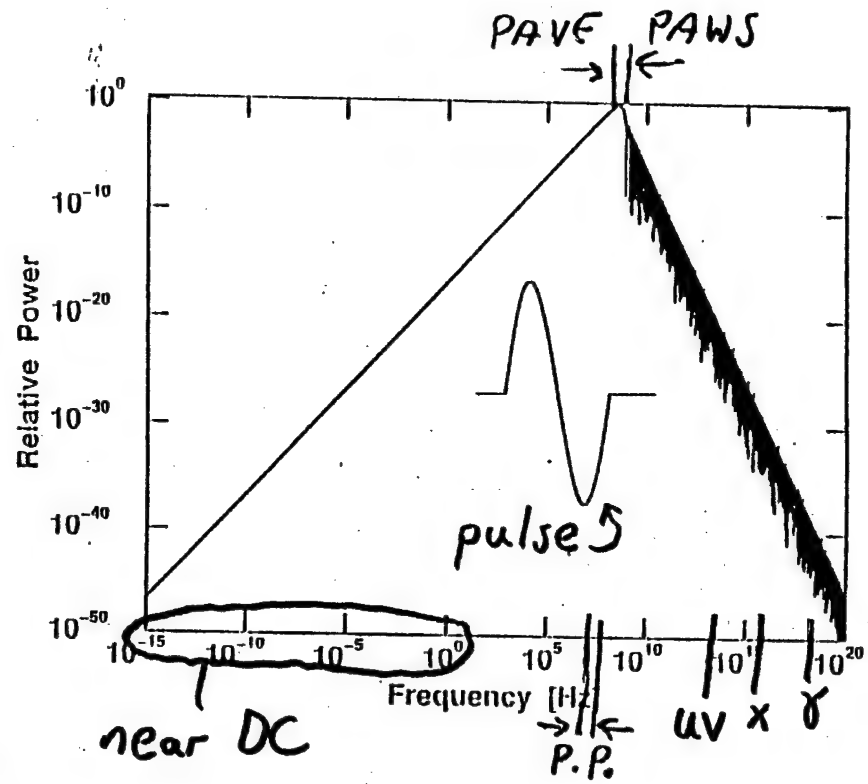
Arithmetic Freq. Scale



- Near DC
- Ultraviolet
- Xrays
- γ rays

} Outside PAVE PAWS Band

Geometric Freq. Scale



Fourth Transparency
Inside and Outside the PAVE PAWS Band

Narrative

I will now give examples of two types of pulses: One that PAVE PAWS could produce, at least in principle. Another, that Brillouin studied in 1914 and that many others later studied, and which PAVE PAWS could not produce.

My first example had to be confined to the PAVE PAWS spectrum. I chose this pulse [left]. It's a synthetic, purely mathematical function that, in a better graph, resembles a carrier frequency with amplitude modulation that dies off quickly. Here, time is along the horizontal axis of the pulse [inset].

I also computed and graphed the power spectrum of this pulse. I will now explain what a power spectrum is. As I've mentioned, there's a relation between a time-dependent pulse and the spectrum of that pulse. This mathematical relation involves the concept of a Fourier transform. Loosely speaking, a pulse is composed of all of its frequency components, with one component for every frequency in the spectrum. This graphed pulse, for example, is composed of one frequency component for each frequency between 420 and 450 MHz. The whole pulse is a sum, in the Fourier sense, of all these frequency components. Finally, the power spectrum is the amount of power in each frequency component, relative to the power in the strongest frequency component. This curve [left] is a frequency-dependent power spectrum. The graph shows that the power at $\approx$ 420.5 MHz is about one-tenth of a part-per-million of the power at 435 MHz in this one example. Please notice the scales on the graph. The frequency scale is linear scale because it's so narrow: It's the PAVE PAWS band. The power scale is logarithmic because of the precipitous drop-offs near the edges of the band. The power is infinitesimal near those edges. Mathematically, in this one example, the power is precisely 0 for 420 MHz on down to DC. It's also 0 from 450 MHz on up to ∞. Thus, the example pulse has a spectrum confined to 420–450 MHz. It can at least in principle be produced by PAVE PAWS. But I don't know whether PAVE PAWS could really produce this pulse.

Here on the right is a 1-cycle sine pulse. It's 0 for awhile, then it's one cycle of a 435 MHz sine, then it's 0 again. I computed this power spectrum in the same way as on the left. I graphed it on the same logarithmic scale. It turns out that this 1-cycle sine, like the 2-cycle sine and the 3-cycle sine and every integer-cycle sine, has a power spectrum that fills the entire electromagnetic spectrum except for DC and ∞. I therefore used a logarithmic scale in frequency. I couldn't use a linear scale, as on the left, because the band here on the right is far too broad. I've marked roughly where the PAVE PAWS spectrum is. The PAVE PAWS spectrum supports a significant part of this example pulse [on the right] because I chose the 1-cycle carrier frequency to be the centerpoint of the PAVE PAWS spectrum. But if one re-graphs this to focus near the PAVE PAWS spectrum, then one will see that a significant part of the example pulse is outside the PAVE PAWS band. That is, a significant part of the 1-cycle sine cannot be produced by PAVE PAWS.

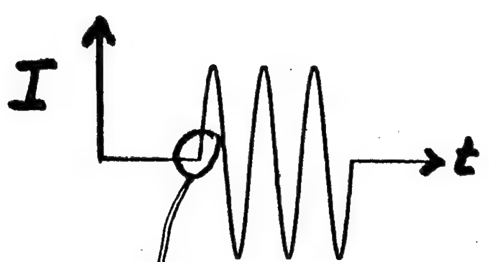
This is why a 1-cycle sine is not a PAVE PAWS-type pulse. The 1-cycle sine has other non-PAVE PAWS qualities associated with some of its tiny components. I mention these because even tiny components have entered public discussions of PAVE PAWS. Here you see that the 1-cycle sine has tiny ultra-violet, x-ray, and gamma-ray components that simply are not emitted by PAVE PAWS or by any other antenna where microwave currents travel around metal. Of course PAVE PAWS can't produce these components: But here they are in a 435-MHz, 1-cycle sine, and in any integer-cycle sine pulse as well. All of the integer-cycle sines also have these near-DC components that PAVE PAWS can't produce because near-DC components are hundreds of MHz removed from the PAVE PAWS band.

N-cycle Sine Currents in Circuits?

(Answer below)

N-cycle sine E field unrealistic for P.P. $\because$ DC $\leq$ freq. $\leq \infty$

Can a circuit produce an N-cycle sine current?



This kink & College L-R-C circuit theory $\Rightarrow$ Physically Impossible

$\boxed{\text{ANSWER = No.}}$

$\therefore$ N-cycle sine currents can't be fed into antennas.

19
19

Fifth Transparency
N-cycle Sine Currents in Circuits?

Narrative

[The supplement for this transparency has an erratum notice below.]

I want to look one level more deeply at the integer-cycle sine. After all, PAVE PAWS might be confined to its 435 MHz ±3.5% band merely for reasons related to spectrum allocation as regulated by the FCC or other agencies. Actually, I don't know why PAVE PAWS is confined to that band. But I wanted to find out whether one could get some type of non-PAVE PAWS antenna to produce an integer-cycle sine. How would one do this? A good start would be to feed an integer-cycle-sine current into an antenna and see whether it radiates an integer-cycle-sine electric pulse. But can one feed an integer-cycle sine current into an antenna? The feeding of a current into an antenna has to be done through a circuit. My rhetorical question finally becomes: Can one produce an integer-cycle-sine current in a circuit?

As an example, I graphed a 3-cycle-sine current. It's 0 for awhile then, as you can see, it suddenly has a kink. A kink is a discontinuity of the slope of the graph. Here [left] the slope is 0 and then suddenly, at the kink, the *slope* jumps to a nonzero value. After this first kink there are 3 cycles of oscillation, and then another kink at the end of the pulse.

The circuits that are used to feed currents into things are called L-R-C circuits. Undergraduates in electrical engineering and physics are taught an equation that represents L-R-C circuits. It turns out that if one takes a current like this one and plugs it into the L-R-C equation, then a little calculus will show that to produce the kink one would have to apply voltages that are discontinuous in time. But voltages that are discontinuous in time don't really exist because real switches and real batteries are neither instantaneous nor ideal. Therefore it's physically impossible to produce a current whose graph has a kink.

Not only are integer-cycle-sine electric-field pulses unrealistic for PAVE PAWS, but one can't even form a current that has an integer-cycle-sine graph.

Supplement

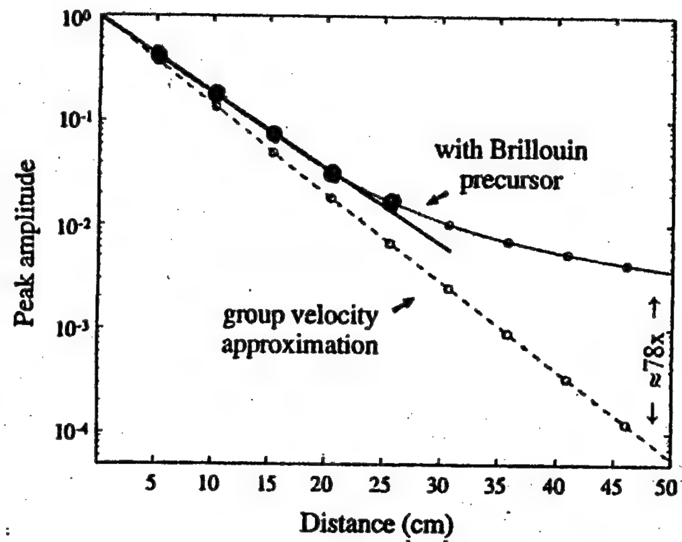
Erratum: I erred during my talk when I spoke of infinite voltages instead of voltages that are merely discontinuous in time. This mistake made the case against kinks in currents seem one-degree stronger than it really is. I apologize for my mistake and its rhetorical consequence. My narrative (above) appears in corrected form. The conclusion that graphs of real currents cannot have kinks does hold in the corrected narrative, as it did in my original presentation. The information on my original transparency also is correct.

The L-R-C circuit equation is $V(t) = L\ddot{Q} + R\dot{Q} + Q/C$ where V is the applied voltage, Q is the charge in the capacitor, and the other letters represent

circuit elements. If the current $\dot{Q}$ has a discontinuous slope (a kink), then $\ddot{Q}$ is itself discontinuous but Q is continuous. These combined circumstances imply that V is discontinuous.

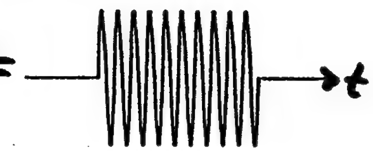
EXAMPLES OF COMPLICATED DECAY

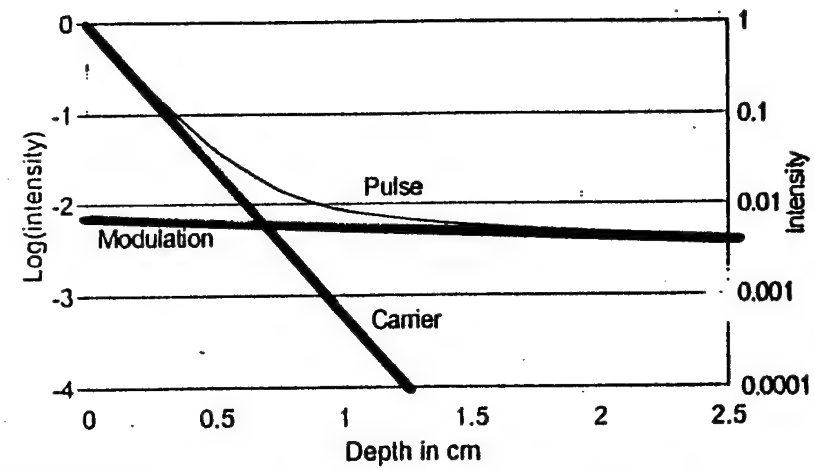
Oughstun 2002
Simulated brain
incident pulse ?



Exponential Decay Until
≈ 1.8% amplitude
≈ 0.03% peak power
(300 ppm)

Adair 1995
simulated water

inc. pulse = 



Exponential Decay Until
≈ 0.3% energy (I showed);
then algebraic decay

Sixth Transparency

Examples of Complicated Decay

Narrative

These are the first numerical results on decay rates that I'll show you. These decay rates are complicated. The next transparency has other numerical examples, but they will involve simpler decay rates of the type discussed in my July 4 paper.

[This report's sixth transparency differs from my presentation's only in the size of 5 numerical data points and the thickness of a solid, straight segment in Prof. Oughstun's graph. Here I enlarged the points and thinned the segment only for graphical clarity.]

This graph [left] is from Prof. Oughstun's interview this year. It regards numerically simulated brain tissue. He didn't say which model he used to simulate the brain. Although he talked about an incident pulse and mentioned its carrier frequency, he didn't mention other features of the pulse. Because the model and the pulse weren't well defined in the interview, the result is irreproducible for now. It was only an interview, after all. But it was interview that included this graph, which is a form of prepared speech. To produce this graph he numerically propagated whatever the pulse was through whatever model he used to simulate the brain. He reasonably assumed that the pulse had the peak value 1 just inside the brain, and he used dots to graph the peak value of the numerically computed pulse every 5 cm in the brain, down through 50 cm of that brain. He connected those dots with a solid curve.

The dashed curve [left] doesn't represent a numerical computation. It represents an approximation that Prof. Oughstun doesn't like. I won't mention that dashed curve again.

Returning to the peak values: They were graphed on a logarithmic scale, as a function of depth on a linear scale. Within a minute or two of my seeing this in May 2002, I made a photocopy and used a ruler and a red pen to draw a line segment through the first 5 data points on the graph. I did this because a straight segment on a linear-log graph turns out to represent exponential decay. The decay beyond 20 cm is clearly slower than the exponential decay from 0–20 cm. But beyond 20 cm the decay is clearly faster than a hypothetical rate characterized by one-quarter the exponent from the first 20-cm of decay. Of course I can't really know whether the decay beyond 20 cm is exponential or algebraic or what-not because the interview's graph is an irreproducible result.

It is clear that there was exponential decay for the first 20 cm of propagation, at which point [20 cm] the amplitude was 1.8% of what it was just inside the brain. At that same depth the peak power, which is another way to measure the size of the pulse, was only 300 ppm (0.03%) of what it was just inside the brain. Thus, there is exponential decay until the pulse becomes small by two measures of size, then the pulse decays in an irreproducible manner.

Prof. Oughstun's result has precedent 7-years earlier in a paper by Prof. Adair.

Adair was commenting on a 1994 paper by Albanese, MD, *et al.* Adair first reproduced a computation by Albanese. They both studied water: Albanese stating his model precisely; Adair, not specifying his model except to say that he and Albanese's eventual numerical agreement was evidence that their two models for water were close. They did use the same incident pulse, namely a 10-cycle sine with a 10 GHz carrier frequency. Adair remarked that he and Albanese had roughly the same results for the peak values, which verified their two numerical methods. Adair then pointed out physical mechanisms affecting the decay of energy, which he referred to as intensity. By stating those mechanisms, he added insight to a problem that had none before. Adair graphed energy on a logarithmic scale and depth on a linear scale. His paper showed how the 10 GHz carrier frequency explained why the initial exponential decay of energy had the slope shown here [right]. His paper specified the relationship. The pulse's rate of decay then slowed down. (All this, so far, would be echoed 7-years later in Prof. Oughstun's graph, evidently for a different model.) Then Adair's pulse began approaching a different rate of exponential decay, which Adair's paper related to the duration of the amplitude-modulated incident pulse in a way that made perfect sense. Thus, Adair showed that there would first be one exponential decay and then a slower exponential decay: All this in just 2.5 cm of water.

A year or two ago I extended Adair's computation up to over 2 meters in water. Adair had noted that his computation through 2.5 cm was trivial; my computation through 2 m also was trivial. I plan to give you details within a month or two. [The details are now in the supplement.] Adair and my computations agree for all depths that he considered. But I monitored the local exponential and algebraic rates of decay through 2 m and found that exponential decay predominated through about 5–10 cm in water, and that algebraic decay predominated from about 15–20 cm through 2 m of water. The result is that exponential decay predominated until the energy had decreased to about 0.3% of its value just inside the water; after that, algebraic decay predominated through over 2 m. These 5–10 cm and 15–20 cm ranges that I mentioned are, to some extent, matters of judgment. But for any reasonable judgment, Prof. Oughstun's result and my extension of Adair's result have similar conclusions.

The conclusion is that Oughstun and Adair's examples show two pulses that first decay exponentially until each pulse is a small fraction of its size just inside the material. This is illustrated for each of three measures of pulse size — peak value, peak power, and energy. That sounds more cut-and-dried than it really is. Really, only Adair's numerical example is reproducible. And even the reproducible example has exponential decay that eventually yields to algebraic decay for unknown reasons. The reproducible example has an integer-cycle sine pulse, whose near-DC content may suggest algebraic decay at large depths; but there is no proof that this would occur. We do have two examples in which pulses decay exponentially until, by any measure, they are small. But it's not known what happens next.

The next transparency will show that the decay is much simpler and much

more well-defined when the spectrum is separated from DC, as in PAVE PAWS.

Supplement

Prof. Oughstun did not respond to my presentation of this transparency. Relevant circumstances are mentioned at the end of the supplementary information for my talk's second transparency.

For Albanese and Adair's example, Fig. 1 shows the energy, relative to its value just inside the water, as a function of depth through 2 m. The relative energy is defined to be 1 at $x = 0$. Straight segments on a semi-log graph, such as the shallow-depth ($x < 1$ cm) segment Fig. 1, are evidence of exponential decay; but when the slope becomes shallow then exponential decay is not as strongly evident. The slope in Fig. 1 is steep for the first 10 cm and it's much shallower beyond that. Drawing tangents to the curve on Fig. 1 also suggests that the decay is non-exponential from around 10 cm to nearly 1.5 m.

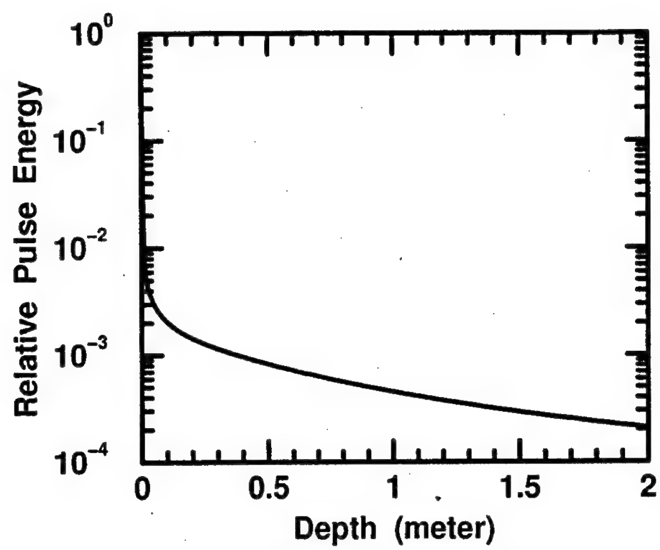


Fig. 1. Semi-log graph of the relative energy of Adair and Albanese's pulse in water. The slope is proportional to the depth-dependent exponential rate of decay. Relative energy = 1 at $x = 0$.

Fig. 2 shows the same relative-energy function as shown in Fig. 1, but this time on a log-log scale. Straight segments on a log-log graph denote algebraic decay at a constant rate. The slope in Fig. 2 is never extremely steep nor, for $x \gtrsim 1$ mm, is that slope extremely shallow. This observation signifies significant algebraic decay for $x \gtrsim 1$ mm. But Adair's graph shows that exponential decay predominates for the first 2.5 cm. This is not a contradiction. The overall decay for the first 2.5 cm is merely more-accurately described by an exponential.

I looked at the x-dependent exponential and algebraic rates of decay. These

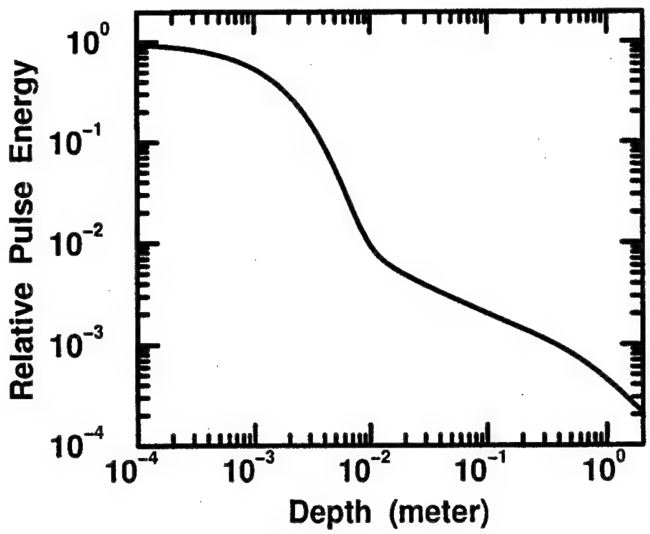


Fig. 2. Log-log graph of the relative energy of Adair and Albanese's pulse in water. The slope is proportional to the depth-dependent algebraic rate of decay.

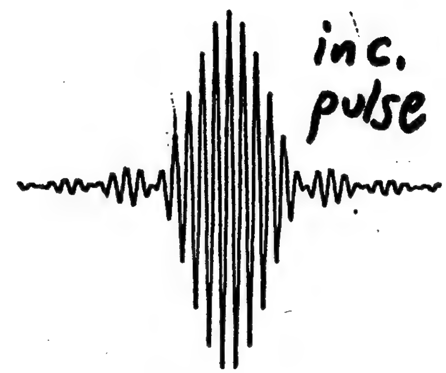
rates are proportional to, respectively, the x-dependent slopes of Figs. 1 and 2. From 10–50 cm, energy decays at an algebraic rate that varies from $x^{-1/2}$ to $x^{-3/4}$. The same energy decays at an exponential rate that varies from 10^{-5x} to $10^{-3x/2}$ over the same 10–50 cm interval. Because it's typical for the exponential decay rate to quickly vanish when the decay is actually algebraic (To see this, compute the algebraic and exponential rates of decay of the pure functions $x^{-1/2}$ and 10^{-x}.), it follows as a matter of judgment that algebraic predominates from 10–50 cm. A similar analysis shows, also as a matter of judgment, that algebraic decay also predominates from 50–200 cm. The location at which the decay becomes predominantly algebraic is again a matter of judgment. I think that any reasonable analysis would conclude that the decay becomes predominantly algebraic near 10–20 cm

EXPONENTIAL DECAY IN A STRONG SENSE

Inc. pulse band $0 < f_{min} \le f \le f_{max}$

Each freq. component decays as

$$A_f(x) = A_f(0)\, 10^{-\underbrace{\alpha(f)}_{\text{measured} > 0 \text{ (lossy)}} \cdot x}$$



Let $\alpha_{min} = MIN(\alpha)$

Then $\mathcal{P}(x) \le \mathcal{P}(0)\, 10^{-\alpha_{min} x}$ (pulse peak)

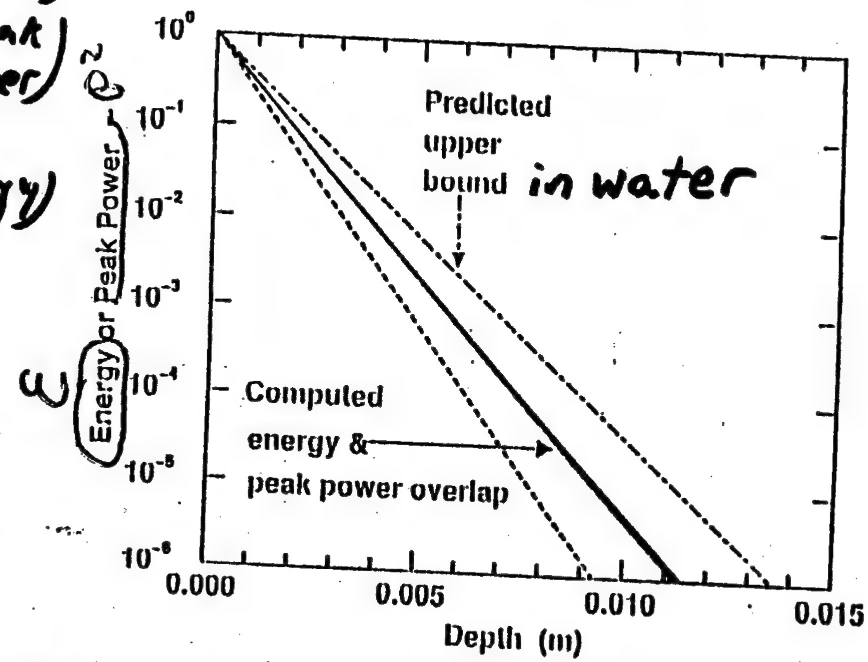
$\mathcal{P}^2(x) \le \mathcal{P}^2(0)\, 10^{-2\alpha_{min} x}$ (peak power)

$\mathcal{E}(x) \le \mathcal{E}(0)\, 10^{-2\alpha_{min} x}$ (energy)

My paper has

4 examples in water

4 examples in Brillouin spectrum of a Lorentz model



Seventh Transparency

Exponential Decay in a Strong Sense

Narrative

Here I will show the results of my July 4 paper in symbols. I will go through the intuition that led me to expect that the first inequality [$\mathcal{P} \leq \ldots$] would be true. Then I will mention the proof and show a numerical example. I will show that each of three measures of pulse size is less than or equal to a pure exponential. Being less than or equal to a pure exponential is simple, it is exponential decay without equivocation, and it always occurs in lossy materials when the incident pulse spectrum is separated from DC, as in the PAVE PAWS case.

To start, my paper assumes that the incident pulse has a spectrum from f_{min} to f_{max}. That is, the pulse is composed of frequency components at all frequencies f from f_{min} to f_{max}. I also assume that $f_{\text{min}} > 0$. That's what I mean by "separated from DC." This condition $f_{\text{min}} > 0$ is the only assumption I make about the incident pulse.

Please consider the frequency components that compose the pulse. Undergraduates in electrical engineering and in physics are taught that each frequency component decays exponentially. The exponential decay rate [here $\alpha(f)$] is a function of the frequency. So the amplitude [$A_f(x)$] of a frequency component at a depth x equals what there was just inside the material [$A_f(0)$] times a decaying exponential [$10^{-\alpha(f)x}$], where this coefficient of exponential decay [$\alpha(f)$] is a function of frequency. This representation for frequency components [$A_f(x) = A_f(0)10^{-\alpha(f)x}$] is useful because there are tables of exponential-decay coefficients for many materials. Sometimes these coefficients are tabulated directly; other times, they're easy to infer from different tabulated coefficients such as the real and imaginary parts of complex permittivity. That is, the $\alpha(f)$ coefficients are either directly tabulated or they are essentially tabulated for many materials. I have a report in my office that tabulates such properties for a dozen or so biological tissues in frequency bands that span the PAVE PAWS band. It turns out that all ordinary materials have exponential-decay coefficients that are strictly positive [> 0] for nonzero frequencies. These are called lossy materials. Biological tissues are lossy. Thus the second assumption of my paper is merely that the material is lossy.

My paper has two assumptions: The pulse has $f_{\text{min}} > 0$, and the material is lossy. There are no more assumptions.

In this case the pulse is composed of all frequency components from f_{min} to f_{max}. Each pulse component decays exponentially at a frequency-dependent rate. Math suggests that, among all these rates of exponential decay, there has to be a *smallest* rate of exponential decay: call it α_{min}. That's useful because every frequency component has to decay at least as fast as that smallest rate of exponential decay. The entire pulse is composed of things that all decay at least as fast as α_{min}. When every part decays at least as fast as α_{min} then, intuitively, the whole pulse should decay at least as fast as α_{min}. That explains why I thought

that the pulse peak [$\mathcal{P}(x)$], relative to its value [$\mathcal{P}(0)$] just inside the material, would decay at least as fast [$\leq$] as the smallest rate of exponential decay [$10^{-\alpha_{\min}x}$]. It seemed intuitively obvious to me.

Intuition is useful because it shows that something makes sense. But intuition does not establish truth to the degree that a derivation does. The derivation of the first inequality is in my paper. It's the first 5-step calculus derivation in my hand-out for the committee. The derivation requires specialized knowledge of integral calculus and of the complex-valued exponential function. Many undergraduates could understand that derivation. It may set a record for simplicity of a derivation of a result central to a peer-reviewed paper on electromagnetic theory.

Thus, I derived that the peak value of a pulse, relative to its peak value just inside the material, is less than or equal to a pure exponential. This is a simple type of decay, unlike on the previous transparency.

Likewise, the peak power and the energy, relative to their values just inside the material, are bounded above by pure exponentials.

I tested those $\leq$-pure-exponential predictions using a model for water specified in my July 4 paper. I used an incident pulse [shown here] that was separated from DC. I didn't know when I wrote the paper in February that exponential decay would be an issue for PAVE PAWS. That's why I chose a pulse that had $f_{\min} = 13.5$ GHz and $f_{\max} = 18$ GHz instead of the PAVE PAWS band. But this pulse fundamentally resembles a PAVE PAWS pulse in that it is separated from DC.

I looked at all the exponential-decay coefficients for water in the frequency range from 13.5–18 GHz. I found the smallest coefficient, and I called it $\alpha_{\min}$. Then I graphed $10^{-2\alpha_{\min}x}$ on this graph whose energy-or-peak-power axis is logarithmic and whose depth axis is linear. This $10^{-2\alpha_{\min}x}$ yielded the dot-dashed straight segment — the highest straight segment — on the semi-log graph. This prediction involved only the decay coefficients of water and the 13.5–18 GHz spectrum; it was independent of any other detail of the incident pulse.

Then I numerically propagated the incident pulse through water and graphed its energy and peak power as both those quantities decayed by a factor of 1 million. A 1-ppm decay is a large decay, I think. Throughout this decay the energy and peak-power graphs almost overlapped. They didn't exactly overlap; in fact, they diverged slightly as depth increased. That explains [in part] why the solid segment is thicker near its end. [The caption of my paper's Fig. 2 explains the thickening more fully but implicitly: That figure really has 4 overlapping curves.] I had not predicted this overlap; it's coincidental as far as I know. Of course I did predict that the energy and peak power would, for every depth, be $\leq$ their dot-dashed upper bounds. You can see here that they are, at least for depths going down to about 1 mm. A blow-up of the first 1 mm of the graph verifies my predicted upper bounds for the first 1 mm as well. Although this verifies two of my predictions, I say that it's only one verification because it's only one pulse that I propagated. My paper has 7 more verifications, 4 of which are in the Brillouin spectrum of a

Lorentz model. So there's nothing special about the Brillouin precursors: They decay exponentially, too, if they are produced by pulses separated from DC. Any Brillouin precursor produced by PAVE PAWS would decay exponentially.

I also want to note that the apparent straightness of the solid curve is evidence that the pulse decays at an exponential rate that is nearly constant. I had not predicted this near constancy. But my paper does study the near-constant exponential decay rate further to establish exponential decay definitively in the numerical examples. This was already established definitively by the theory, but definitive numerical examples also seem worthwhile.

Supplement

Prof. Oughstun did not respond to my presentation of this transparency. Relevant circumstances are mentioned at the end of the supplementary information for my talk's second transparency.

There was a question or statement from a committee member about whether I made standard use of the word "lossy." This is covered below in the "Intuition for Algebraic Decay" part of the "Answers to Questions Asked During my Talk" section of this report. To be clear about my usage: I say that a material is lossy if and only if (iff) $\mathrm{Im}k > 0$ for all frequencies of interest. I say that a material is passive iff $\mathrm{Im}k \geq 0$ for all frequencies of interest. Thus, lossy materials are passive. I say that any other material is active. That is, I say that a material is active iff $\mathrm{Im}k < 0$ for at least one frequency of interest.

THE WORD "PRECURSOR"

- "Precursor" has a widely accepted meaning only for pulses in Lorentz models.

  Original intent, 1914: N-cycle sine pulse

  Precursors = $\left\{ \begin{array}{l} \text{Wave components that arrive} \\ \text{before the carrier} \end{array} \right\}$

  Here, precursors are out-of-band by def.

- No definition of "precursor" is used by more than a few people, except when used for Lorentz models.

- A mere name has no scientific significance.

  "Precursors" in a new context don't necessarily inherit behavior from "precursors" of another context.

31
31

Eighth Transparency

The Word "Precursor"

Narrative

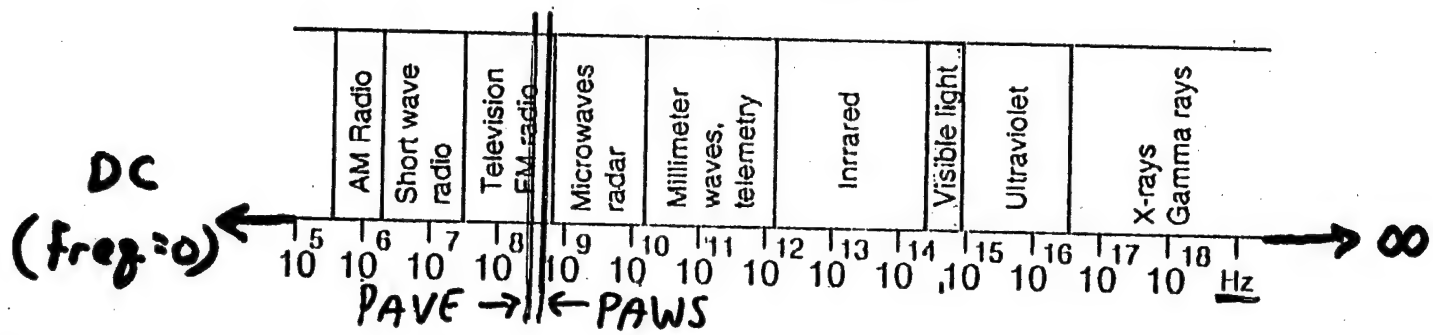
I don't have time for all the transparencies. I apologize.

Supplement

The transparency is self-explanatory. I have seen the word "precursor" used outside the context of Lorentz models. In those cases, no single definition of "precursor" was used by more than a few people.

The transparency's final item ("A mere name...") seems obvious. The transparency includes that item only because I actually reviewed a journal manuscript that used mere linguistic coincidence to claim that so-called precursors in Debye models would inherit the properties of Lorentz-model precursors. The manuscript that contained that claim was eventually withdrawn by its author.

SUMMARY



- N-cycle sines unrealistic for P.P. pulses & currents
- Slower-than-exp. decay ⇒ DC or near-DC content. (co-ax cables or near antennas)
- Outdoors, far from antennas, spectra separated from DC. Pulses decay as $\leq 10^{-\alpha_{min} x}$ in each lossy material.

- Thanks to Hans Steyskal for useful conversations.

Ninth Transparency
Summary

Narrative

It's time to sum up.

Air Force documents from 1980 and 2002 say that the PAVE PAWS spectrum is 420–450 MHz. The bandwidth is $\pm$ 3.5% of the center frequency. This is the tightest restriction I know of on what PAVE PAWS can broadcast. It is evidently the definitive restriction on pulse shapes.

Integer-cycle-sine electric pulses are unrealistic models for PAVE PAWS pulses because integer-cycle sines are not confined to the PAVE PAWS spectrum. Integer-cycle sines pulses are supported over the entire electromagnetic spectrum, except for the frequencies that are precisely 0 or ∞. Integer-cycle-sine currents, as distinct from pulses, cannot even be produced in a real circuit. So much for integer-cycle sines.

Prof. Oughstun expressed concern, in his interview, about slower-than-exponential decay in lossy materials. But, as I mentioned, if the spectrum is separated from DC then the decay is exponential in lossy materials. It follows that Prof. Oughstun is concerned about pulses that have DC or near-DC content that PAVE PAWS cannot produce; after all, the PAVE PAWS spectrum is separated by hundreds of MHz from DC. Prof. Oughstun's concern is therefore irrelevant to PAVE PAWS pulses.

Prof. Oughstun's statement is relevant to some non-PAVE PAWS structures, co-axial cables for example. If you have cable TV then you probably have a coax cable that runs between the wall and the TV. That coax cable could transmit DC and near-DC content pulses to your TV. Aside from that example, a current source could produce DC or near-DC content pulses that can be seen if you're near the current source. Antennas have current sources, so you may be able to see those type of pulses near antennas. The simplest example I can think of involves a capacitor. A capacitor is a device for storing charge. If you charge up a capacitor and use a resistive wire to connect it to ground, and then flip a switch to start the current, the electric field near the wire will have DC content. The DC content will no go far, but it will be present near the wire. So if you're near a current source, or if you're in a coaxial cable, you can get the type of pulses that concern Prof. Oughstun. In a related matter, I showed that all Prof. Oughstun's Olver-method work is for DC and near-DC content pulses. That work applies if you live in a coaxial cable or another guided-wave structure, or if you're near a DC-content current source.

But outdoors, far from antennas, pulse spectra are typically separated from DC. My July 4 paper mentions reasons for this, involving radiation efficiency of small dipoles and loops. There's even a stronger case for PAVE PAWS being separated from DC because PAVE PAWS is tuned to 400 MHz or so. It's not tuned to DC. And the resulting impedance mismatch at low frequencies makes it

even harder to broadcast near-DC content than the inefficiency of small antennas suggests. In any case, whenever the spectrum is separated from DC, then all three standard measures of pulse size will be $\leq$ a pure-exponential decay in each lossy material.

That covers the content of my talk.

I want to add that my degree is in mathematical physics. I am not an engineer. Whenever I have a question about antennas, I just walk down the hall and someone or other will tell me the answer. In this way, Hans Steyskal and I conversed for many hours while I was preparing this talk. These conversations were far more useful to me than they could have been to him. I thank Hans for that. And I thank you for your attention to my talk.

Supplement

Prof. Oughstun did not respond to my presentation of this transparency. Relevant circumstances are mentioned at the end of the supplementary information for my talk's second transparency.

A committee member mentioned an IEEE standard for fields near antennas. For the PAVE PAWS frequency band, the member said that a normal person could stand at the antenna and spit beyond the corresponding near field. This was said in context of a discussion of being near a DC-content current source.

Justification for Executive Summary

This section will justify, in order, the Executive Summary's first two paragraphs and its two lists.

The first paragraph's statement about the PAVE PAWS band is from Air Force reports cited in the supplement for my first transparency.

The second paragraph summarizes the second list, numbered [1]–[6].

I will now justify the list numbered (1)–(12). Item (10) is justified in my third transparency. The paragraph that contains (7.3.4) on p. 277 of Oughstun and Sherman's book begins, "For the examples considered in this research..." That quotation implies that pulse types that I list as (1)–(4) and (7)–(9) are examples of my list's item (10). This is clear from the context of the book's pp. 159, 275–280, 351 and 352; and in context of the central role in the book of Olver's method from Sec. 5.1 (pp. 159*ff*); and in context of the pulse types repeated in the book's Table of Contents for Subsections 4.3.1–4.3.4 and 7.2.4–7.2.7 and 7.3.4–7.3.7 and 8.3.1–8.3.4 and 9.4.1–9.4.4. One may also introduce phase changes into exponents to switch from amplitude-modulated sines to amplitude-modulated cosines. It is a straightforward but tedious exercise to calculate the closed-form expressions for the Fourier transforms of pulse types (3) and (4), and thereby verify that those types have DC or near-DC content.

The first list is now justified except for items (5), (6), (11), and (12). This paragraph deals only with (5) and (6). Pulse types (5) and (6) have DC or near-DC content, as follows from IS Gradshteyn and IM Ryzhik, and A Jeffrey ed., *Table of Integrals, Series, and Products* 5th edn. (Academic, San Diego, Calif., 1994) [Engl. transl. of *Tablitsy Integralov, Summ, Riadov i Proizvedeniĭ*], Eqs. 3.896.4 and 17.23.19, and Sec. 17.21. If the Fourier transform of f exists then its $\omega = 0$ value is 0 iff $\int_{-\infty}^{\infty} f(t)\mathrm{d}t = 0$. Also please notice that $\exp(-t^2/\tau^2)\cos(\omega_c t)$ is an even function of t.

The first list is now justified except for items (11) and (12). Pulse type (11) has DC or near-DC content by definition. Pulse type (12) has DC or near-DC content, as shown by my July 4, 2002 paper, which is included in this report.

By reading all material referred to in items [1]–[6], one can determine that the collective work is devoted largely or entirely to pulse types (1)–(12). I have read all that material. Item [5], for example, includes nearly 2000 pages.

Answers to Questions Asked During my Talk

I was asked about the following matters.

1. *Intuition for Algebraic Decay*
2. *An Incorrect Claim Regarding Exponential Decay*
3. *Vita and Publications*

I will answer here more fully than I did during my talk.

1. *Intuition for Algebraic Decay*

A committee member asked for my intuition regarding the origin of algebraic decay. I will sketch this intuitive origin below. I developed this exact intuition in 1991, but I never published it. I apologize for not having remembered the following ideas when the committee member asked for them.

A frequency component is represented by $\exp(ikx)$ where the wavenumber $k \propto f\sqrt{\varepsilon(f)}$, f=frequency, and ε is the complex-valued permittivity of a model or material. The decay rates of frequency components regard only the magnitudes

$$|\exp(ikx)| = \exp(-\mathrm{Im}[k(f)]x). \tag{1}$$

If $\varepsilon \neq \infty$ at DC, as is true for all Lorentz and Debye models, then $k = 0$ at DC. Figure 3 sketches an example of this behavior.

Notice in Fig. 3 that the imaginary parts $\mathrm{Im}[k(f_j)]$ decrease as the frequencies f_j decrease toward DC. Thus, for frequency components in the interval [DC, f_1], the higher frequencies will be attenuated more severely than the lower frequencies in the model of Fig. 3. This is called a low-pass filter because the low-frequency components pass through the model more readily than the high-frequency components.

Let's now make a nonrigorous, but instructive, leap: Imagine that the initial pulse fills the frequency band [DC, f_1]. Then the steepest attenuation is characterized by a relatively high exponential rate $\mathrm{Im}k(f_1)$. As the pulse propagates, attenuation decreases until the spectrum now fills only [DC, f_2] and the decay is characterized by a slower exponential rate $\mathrm{Im}k(f_2)$. As this nonrigorous, intuitive process continues for a sequence of frequencies $f_1, f_2, f_3, \ldots \to 0$, the characteristic rate of exponential decay will go as $\mathrm{Im}k(f_1), \mathrm{Im}k(f_2), \mathrm{Im}k(f_3), \ldots \to 0$. That is, as the pulse propagates infinitely far, its decay rate decreases until the decay is slower than any exponential rate. Thus the property $k(0) = 0$ for nonsingu-

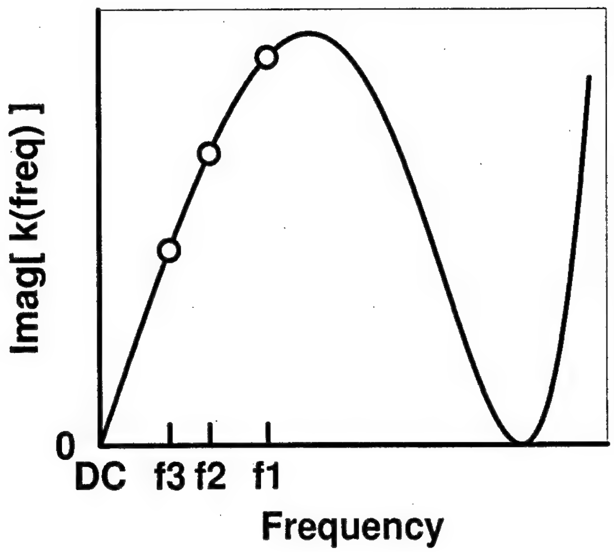


Fig. 3. Idealized sketch of a function $\mathrm{Im}[k(f)]$ that has two 0 values.

lar ($\varepsilon \neq \infty$ at DC) models should yield slower-than-exponential decay for any pulse that initially has DC or near-DC content. Algebraic decay could be such a decay. Although there are, in principle, decay rates that are faster than algebraic and slower than exponential, none of these has arisen from analysis of pulse propagation in any paper that I know of.

Now please examine the second 0 of $\mathrm{Im}[k(f)]$ in Fig. 3. If a pulse has spectral content near (but not at) that second 0 then slower-than-exponential decay, perhaps even algebraic decay, could be expected. But if $f_{\mathrm{null}} > 0$ is that second 0 — that is, if $\mathrm{Im}[k(f_{\mathrm{null}})] = 0$ — then a steady-state wave with frequency f_{null} would propagate unattenuated through the model. Because it seems intuitively obvious to many physicists that real materials have nonzero loss for all nonzero f, I excluded the possibility of a second 0 of $\mathrm{Im}k$ from my *Electronics Letters* paper.

For completeness, I should clarify my paper's unstated justification for using $\mathrm{Im}k > 0$ to represent lossy materials. The justification is in Jackson's 2nd edn. of *Classical Electrodynamics*. There, please start reading with the sentence before (7.53), end with (7.54), and include the footnote on p. 286. That text applies even to general dispersion models $k(f)$, not just to so-called resonant models. The condition $\mathrm{Im}k > 0$ is thereby shown to be closely related to the more commonly mentioned "lossy" condition $\mathrm{Im}\varepsilon > 0$ in the usual context of non-amplifying media. [For another example equating "loss" with $\mathrm{Im}\varepsilon > 0$, read p. 274 through the sentence that contains (80.7) in LD Landau *et al.*, *Electrodynamics of Continuous Media* 2nd edn. (Pergamon, 1984) Engl. transl. by JB Sykes *et al.*] My definition of loss is therefore equivalent to the ordinary definition of loss when used in my paper's context of $\mathrm{Im}k(f) > 0$ throughout the spectrum of frequencies $0 < f_{\mathrm{min}} \leq f \leq f_{\mathrm{max}}$ of a pulse separated from DC. Defining "loss" this way made my paper briefer, which was essential under the journal's 1.5-page limit.

2. *An Incorrect Claim Regarding Exponential Decay*

Prof. Oughstun's talk presented a model whose complex permittivity satisfied

$$\varepsilon = \varepsilon_{\mathrm{a}} + \varepsilon_{\mathrm{named}} + \mathrm{i}\sigma/\omega, \tag{2}$$

where $\varepsilon_{\mathrm{named}}$ was a named model (I forget the name), $\sigma \geq 0$ and $\varepsilon_{\mathrm{a}} > 0$ were independent of ω, and $\omega = 2\pi\times$frequency. [This section is careless with multiplicative constants because they don't affect this section's results on algebraic and exponential decay.] Prof. Oughstun claimed there was a threshold σ_{thresh} such that the pulse decay was algebraic if $\sigma < \sigma_{\mathrm{thresh}}$, but exponential if $\sigma > \sigma_{\mathrm{thresh}}$. This is my best recollection. I will show here that Prof. Oughstun's claim, as I recall it, is incorrect.

During my talk, two committee members asked about the relation of Prof. Oughstun's claim (above) to my July 2002 paper on exponential decay. I replied that I hadn't seen Prof. Oughstun's new, unpublished claim before, so I could not immediately answer. After a few hours' thought, I have concluded that Prof. Oughstun erred in claiming that $\sigma > \sigma_{\mathrm{thresh}}$ implies exponential decay. This follows immediately from a textbook result, as I will now explain.

Pick a named model $\varepsilon_{\mathrm{named}}$ that yields algebraic decay as in Prof. Oughstun's claim. Pick $\sigma_{\mathrm{thresh}} < \sigma < \infty$ with σ so profoundly large that $|\mathrm{Im}\varepsilon_{\mathrm{named}}|$ is negligible compared with σ/ω of (2) for the frequency band at hand. Similarly, pick ε_{a} large enough that $|\mathrm{Re}\varepsilon_{\mathrm{named}}|$ is negligible by comparison. Neglecting $\varepsilon_{\mathrm{named}}$ and using $\mu = 1$ yields

$$\varepsilon = \varepsilon_{\mathrm{a}} + \mathrm{i}\sigma/\omega, \tag{3}$$

$$k = \mathrm{c}^{-1}\omega^{1/2}\left(\varepsilon_{\mathrm{a}}\omega + \mathrm{i}\sigma\right)^{1/2}, \tag{4}$$

from which one can show that pulses $E(x,t)$ in ε of (2) satisfy the PDE $E_{xx} - \sigma E_t - \mathrm{c}^{-2}E_{tt} = 0$. A change of scale for the variables x and t yields the telegraph equation (5.271) of [Z]=[E Zauderer, *Partial Differential Equations of Applied Mathematics* 2nd edn. (Wiley, 1989) Example 5.15 on pp. 301–307.] [Z] says on p. *ix* that it "is intended for advanced undergraduate and beginning graduate students..."

Suppose that a pulse somehow gets into this model and, at a time called $t = 0$, the pulse profile in the model is smooth in the sense that $E(x,0)$ and $E_t(x,0)$ have continuous first derivatives in x. The conclusion in [Z] from (5.290) through the first full paragraph of p. 306 is that, if the *spacial* Fourier transforms of $E(x,0)$ and $E_t(x,0)$ have nonzero components at $k = 0$, then $E(x,t)$ will propagate with its (x,t)-dependent peak satisfying $x^2 \lesssim 4t$. [Z] adds that the field value at that peak will decay as $t^{-1/2}$ for large t; equivalently, it will decay as $1/x$ for large x. These decay rates are algebraic, contrary to the exponential decay newly claimed by Prof. Oughstun in his talk.

This may seem sufficient, but I will study the matter further.

Please note that algebraic decay arose from the nonzero pulse components at $k = 0$. Because the telegraph equation ((5.271) of [Z]) is linear, it follows that each pulse component is an ω-dependent amplitude times $\exp(\mathrm{i}[k(\omega)x + \omega t])$, with k as in (4). Because (4) shows that $k = 0$ if and only if $\omega = 0$, the nonzero component at $k = 0$ is associated with a nonzero component at DC ($\omega = 0$). In this report's section on the intuitive basis for algebraic decay, it is explained how this circumstance is expected to yield algebraic decay.

Please note also that (4) implies that ε is lossy in the sense that $\mathrm{Im}[k(\omega)] > 0$ for all $\omega > 0$. My July paper then implies that any pulse with spectrum separated from DC would decay exponentially in ε of (3).

I have just shown that many DC-content pulses will decay algebraically in the large-σ limit (3) of Prof. Oughstun's ε in (2), but all pulses separated from DC will decay exponentially in that limit. I think that this answers the committee members' questions as thoroughly as allowed by my limited memory of Prof. Oughstun's new, unpublished claim.

3. *Vita and Publications*

In the general question-and-answer session, an audience member repeatedly said that he was unable to find my vita or publication list on the web. He seemed puzzled by this. Committee members responded at length, but there was no chance for me to reply.

I am a civilian federal employee. I have never found a civilian federal employee's vita on a workplace web site. I do not have a personal web site nor do I own a computer, although I do specialize in computational applied mathematics at work.

My vita follows. It includes a publication list.

THOMAS M. ROBERTS

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EDUCATION

8/81 to 12/87	INDIANA UNIVERSITY, Bloomington, Indiana Ph.D. in Mathematical Physics — Interdepartmental (1987) Advisor: Distinguished Professor Roger Newton Qualifying Exams: Physics, Mathematics M.A. in Mathematics (1985) M.S. in Physics (1983)
9/77 to 6/81	UNIVERSITY OF CONNECTICUT, Storrs, Connecticut B.S. in Physics Honors Program, Summa Cum Laude Fifteen Graduate Credits in Mathematics & Physics University Scholar: The University's Highest Honor Phi Beta Kappa (junior-year member)

RECENT JOBS

Since 4/96	AIR FORCE RESEARCH LABORATORY, Hanscom AFB, Mass. • Mathematician (GS-13 civilian federal employee) • Conduct & Review Antenna-related Applied Mathematics: I propose and perform independent applied mathematical research; *e.g.*, on reducing antenna interference caused by near-field objects. Using my earlier research, I provided solicited advice to DARPA about antenna placement on Global Hawk platforms. I also review proposals, documents, and presentations.
8/91 to 3/96	AIR FORCE RESEARCH LABORATORY, Brooks AFB, Texas • Mathematician (contractor) • Transient Electromagnetic Propagation and Inverse Scattering: I worked in one Air Force division under several contractual employers. I proposed and performed independent applied mathematical research on the electromagnetics of materials.

EARLIER JOBS

2/89 to 8/91	AMES LABORATORY — USDOE, Ames, Iowa Postdoctoral Fellow in Applied Mathematics (Electromagnetics) Ames Laboratory is a national laboratory, operated for the U.S. Department of Energy by Iowa State University.
10/86 to 2/89	SABBAGH ASSOCIATES, Bloomington, Indiana Senior Technical Analyst Analysis, Numerics, and Collaboration in Electromagnetics
7/85 to 10/86	CONOCO RESEARCH FELLOWSHIP, Bloomington, Indiana Dissertation Research
1/85 to 7/85	SABBAGH ASSOCIATES, Bloomington, Indiana Technical Analyst: Analysis and Numerics in Electromagnetics
8/81 to 1/85	INDIANA UNIVERSITY, Bloomington, Indiana Associate Instructor (23 Months) Research Assistant (18 Months) Service: Curriculum and Exams Committee

COMPUTING SKILLS

Two Semesters Graduate Numerical Analysis: A, A+
Twenty-eight Years Experience: I have supervised four programmers
Workstations and Mainframes, Including Supercomputers
FORTRAN, C, C++, LINUX, WINDOWS, BASIC, VAX/VMS, PL/I, ...

REFEREE EXPERIENCE

National Science Foundation
Air Force Office of Scientific Research
Defense Advanced Research Projects Agency (DARPA)
Small Business Administration (SBIR & STTR)
IEEE Transactions on Antennas and Propagation
Radio Science
SIAM Journal on Applied Mathematics
Journal of the Optical Society of America A
Inverse Problems
Journal of Physics A: Mathematical and General
Journal of Physics D: Applied Physics
Physica B
International Journal of Mathematics and Mathematical Sciences
New Journal of Physics

PAPERS

All papers are available on request. Unwritten presentations, such as seminars and colloquia, are not listed. The principal author is noted in small capitals, for each paper that has a principal author.

$* \Longrightarrow$ Peer-reviewed Paper (published or accepted)
$\circ \Longrightarrow$ Submitted for Peer-reviewed Publication
SMALL CAPITALS $\Longrightarrow$ Principal Author

∘37. T.M. ROBERTS, "Measured and Predicted Behavior of Pulses in Debye- and Lorentz-type Materials," submitted 7/17/02. Now in review.

*36. T.M. ROBERTS, "Radiated Pulses Decay Exponentially in the Far-fields of Antennas," *Electronics Letters* **38** (2002) 679-680.

*35. T.M. ROBERTS, "The Modern Asymptotic Theory: Comment," *Journal of the Optical Society of America A*, accepted for publication 10/31/01. In 2002 the U.S. Air Force gave me a $1,000 award, known as the Chief Scientist's Award, for this paper.

*34. T.M. ROBERTS and P.G. Petropoulos, "Asymptotics and Energy Estimates for Electromagnetic Pulses in Dispersive Media: Addendum," *Journal of the Optical Society of America A* **16** (1999) 2799-2800.

*33. T.M. ROBERTS, "Time-domain Deconvolution Removes the Effect of Near-field Scatterers," *Journal of Computational Physics* **149** (1999) 293–305. There is a related full-page abstract: T.M. ROBERTS, "Use of Time-domain Deconvolution to Reduce Platform-dependent Interference," in 1999 Digest USNC/URSI National Radio Science Meeting, p. 252, July 11-16, 1999, Orlando, Florida.

32. T.M. ROBERTS, "Time-domain Deconvolution Removes the Effects of Near-field Scatterers," Technical Report AFRL-SN-RS-TR-1998-113, ADA361850 (US Air Force, 1998) 18 pages. This provides a proof referred to by my 1999 paper in *Journal of Computational Physics* (above).

31. T.M. ROBERTS, "Electromagnetic Radiation Inverse Scattering," Technical Report TR-1997-0107, AL/OE, ADA340974 (US Air Force, 1997) 120 pages. This is composed of four manuscripts that were funded by a specific government contract. Versions of all the manuscripts were published.

*30. T.M. ROBERTS and P.G. Petropoulos, "Asymptotics and Energy Estimates for Electromagnetic Pulses in Dispersive Media," *Journal of the Optical Society of America A* **13** (1996) 1204–1217.

29. T.M. ROBERTS, "Energy Estimates and Layer Stripping for Depth Dependent, Electromagnetically Dispersive Media" (abstract), in "Inverse Problems Newsletter," *Inverse Problems* **11** (1995) 271–286.

*28. T.M. ROBERTS, "Comment on 'The Nonlinear Operation of a Microwave Crossed-field Amplifier,' " *IEEE Transactions on Electron Devices* **40** (1993) 1188–1189.

27. T.M. ROBERTS, "An Inverse Scattering Algorithm for Layered, Electromagnetically Dispersive Media," in *Second International Conference on Mathematical and Numerical Aspects of Wave Propagation*, R. Kleinman, T. Angell, D. Colton, F. Santosa, and I. Stakgold eds. (Society for Industrial and Applied Mathematics [SIAM], Philadelphia, 1993) Chap. 44, pp. 417–425. A related 1000-word extended abstract appeared on pp. 93–94 of a booklet whose cover reads, "Extended Abstracts, Second International Conference on Mathematical and Numerical Aspects of Wave Propagation, June 7–10, 1993, Clayton Hall, University of Delaware, Conducted by: Society for Industrial and Applied Mathematics with the cooperation of the Institut National de Recherche en Informatique et en Automatique, SIAM, INRIA."

*26. T.M. ROBERTS and M. Hobart, "Energy Velocity, Damping, and Elementary Inversion," *Journal of the Optical Society of America A* **9** (1992) 1091–1101.

*25. T.M. ROBERTS, "Causality Theorems," in *Inverse Problems and Invariant Imbedding*, J.P. Corones, G. Kristensson, P. Nelson, and D.L. Seth eds., (Society for Industrial and Applied Mathematics [SIAM], Philadelphia, 1992) Chapter 9 (pages 114–128). The volume's preface says, "The editorial decision to include each technical contribution appearing in these proceedings was based upon an anonymous review." The revised manuscripts were also reviewed.

*24. T.M. ROBERTS, "Introduction to Schrödinger Inverse Scattering," *Physica B* **173** (1991) 157–165. Also presented as an invited talk to the Workshop on Methods of Analysis and Interpretation of Neutron Reflectivity Data, Argonne National Laboratory (1990).

23. T.M. ROBERTS and M. Hobart, "Elementary Inversion, Energy Velocity, and a Generalized Sommerfeld Theorem," *Digital Image Synthesis and Inverse Optics*, A.F. Gmitro, P.S. Idell, and I.J. LaHaie eds., Proc. SPIE **1351** (1990) 107–116.

*22. T.M. ROBERTS, "Inverse Scattering for Step-periodic Potentials in One Dimension," *Inverse Problems* **6** (1990) 797–808.

*21. T.M. ROBERTS, "Scattering for Step-periodic Potentials in One Dimension," *Journal of Mathematical Physics* **31** (1990) 2181–2191.

*20. T.M. ROBERTS, "Explicit Eigenmodes for Anisotropic Media," *IEEE Transactions on Magnetics* **26** (1990) 3064–3071.

19. J.C. TREECE, T.M. Roberts, D.J. Radecki, and S.D. Schunk, "Detecting Microstructure and Flaws in Composites Using Eddy-current Instrumentation," *Review of Progress in Quantitative Nondestructive Evaluation* **8B** (1990) 1519–1526.

18. T.M. ROBERTS and H.A. Sabbagh, "Review of Nonlinear Analysis for Crossed-field Amplifiers," advisory report prepared under contract with Naval Weapons Support Center, Code 80312, Crane, Indiana 47553 (20 January 1989) 33 pages plus attachment.

*17. T.M. ROBERTS, H.A. Sabbagh, and L.D. Sabbagh, "Electromagnetic Interactions with an Anisotropic Slab," *IEEE Transactions on Magnetics* **24** (1988) 3193–3200.

*16. T.M. ROBERTS, H.A. Sabbagh, and L.D. Sabbagh, "Electromagnetic Scattering for a Class of Anisotropic Layered Media," *Journal of Mathematical Physics* **29** (1988) 2675–2681.

*15. H.A. SABBAGH, L.D. Sabbagh, and T.M. Roberts, "An Eddy-current Model and Algorithm for Three-dimensional Nondestructive Evaluation of Advanced Composites," *IEEE Transactions on Magnetics* **24** (1988) 3201–3212.

14. L.D. SABBAGH, T.M. Roberts, D.J. Radecki, J.C. Treece, and H.A. Sabbagh, "A Computational Model for Electromagnetic Interactions with Advanced Composites," Proceedings of the Fourth Annual Review of Progress in Applied Computational Electromagnetics at the Naval Postgraduate School, Monterey, CA (1988) 10 pages.

13. J.C. TREECE, T.M. Roberts, and S.D. Schunk, "Electromagnetic Imaging for Reconstruction of Flaws in Advanced Composites," *Review of Progress in Quantitative Nondestructive Evaluation* **7A** (1988) 349–356.

12. J.C. TREECE, T.M. Roberts, and S.D. Schunk, "Modeling of Electromagnetic Fields Produced by Eddy Currents in Advanced Composites," presented to Society for Manufacturing Engineers, September 1987.

11. T.M. ROBERTS, "Inverse Scattering in One Dimension, with a Crystal-vacuum Interface," Ph.D. dissertation (1987), available from University Microfilms International, 300 North Zeeb Road, Ann Arbor, MI, 48106.

10. H.A. SABBAGH, L.D. Sabbagh, and T.M. Roberts, "A Computational Model for Electromagnetic Interactions with Advanced Composites," *Review of Progress in Quantitative Nondestructive Evaluation* **6A** (1987) 211–215. This is a concise version of a 1986 proceedings paper with the same title, below.

9. T.M. ROBERTS and H.A. Sabbagh, "A Model for Eddy-current Interactions With Advanced Composites," *Review of Progress in Quantitative Nondestructive Evaluation* **5B** (1986) 1105–1111.

8. H.A. SABBAGH, L.D. Sabbagh, and T.M. Roberts, "A Multifrequency Algorithm for Inverting Eddy-current Data and Reconstructing Three-dimensional Anomalies in Graphite-epoxy," *Review of Progress in Quantitative Nondestructive Evaluation* **5A** (1986) 409–415.

7. H.A. SABBAGH, T.M. Roberts, and L.D. Sabbagh, "A Computational Model for Electromagnetic Interactions with Advanced Composites," Proceedings of the Second Annual Review of Progress in Applied Computational Electromagnetics at the Naval Postgraduate School, Monterey, CA (1986) 43 pages. This paper has the same title as a 1988 paper in the same conference series; however, the papers differ distinctly.

6. H.A. SABBAGH, L.D. Sabbagh, and T.M. Roberts, "An Eddy-current Model for Three-dimensional Nondestructive Evaluation of Advanced Composites," Technical Report NSWC TR 85-304, ADA167688 (U.S. Navy, 1985) 64 pages.

The following papers regard transformation and displacement in general relativistic spaces with torsion. The papers appear almost verbatim in P. Moon and D. Spencer, *Theory of Holors* (Cambridge University Press, Cambridge, UK, 1986) pp. 215–224, 229–232, 234–240, and 253–270. Peer review of the textbook implies peer review of papers used verbatim therein.

∗5. M. Cavagnaro, T. Roberts, R. Semagin, and D. Spencer, "Akinetor Calculi," presented to the American Mathematical Society (AMS), Amherst, MA (1985).

∗4. M. Cavagnaro, T. Roberts, R. Semagin, D. Spencer, and S. Uma, "Akinetors, the Riemann-Christoffel Tensors, and the Ricci Tensors," presented to the AMS, Amherst, MA (1985).

∗3. M. Albert, M. Cavagnaro, T. Roberts, and D. Spencer, "The Bianchi Identities," presented to the AMS, Amherst, MA (1985).

∗2. M. Cavagnaro, T. Roberts, R. Semagin, D. Spencer, and S. Uma, "The Riemann-Christoffel Tensors," presented to the AMS, Worcester, MA (1985).

∗1. M. Cavagnaro, T. Roberts, R. Semagin, and D. Spencer, "Tensor Calculi and the Linear Connection," presented to the AMS, Pittsburgh, PA (1981).

SIGNED, PUBLISHED REVIEWS

Mathematical Reviews is the world's leading reviewer of PUBLISHED mathematical work. I was invited to become one of their reviewers because of a written recommendation from a long-time reviewer. *Mathematical Reviews* sends me PUBLISHED papers to review. Signed, evaluative reviews, such as mine below, are generally published as-is, after copy editing, with the reviewer's name attached. These circumstances denote trust. The reviews themselves are published on paper, on CD-ROM, and by subscription to MathSci on the web (`http://www.ams.org/mathscinet`). My reviews are also available from me.

13. Signed review published as MR 1 872 918 (PRELIMINARY) by *Mathematical Reviews* (2002) for C. Amrouche, R. Luce, and S. Perez, "Identification of the thickness of a thin layer from boundary measurements," *Inverse Problems* **17** (2001) 1703-1716.
12. Signed review published as MR2002a:78039 by *Mathematical Reviews* (2002) for M.S. Min and C.H. Teng "The Instability of the Yee Scheme for the 'Magic Time Step,' " *Journal of Computational Physics* **166** (2001) 418–424. (Reviewed jointly with P. Petropoulos)
11. Signed review published as MR2001h:78044 by *Mathematical Reviews* (2001) for Z. Chen, Q. Du, and J. Zou "Finite Element Methods with Matching and Nonmatching Meshes for Maxwell Equations with Discontinuous Coefficients," *SIAM Journal on Numerical Analysis* **37** (2000) 1542–1570.
10. Signed review published as MR2000g:78012 by *Mathematical Reviews* (2000) for P. Hähner "An Inverse Problem in Electrostatics," *Inverse Problems* **15** (1999) 961–975.
9. Signed review published as MR2000a:78029 by *Mathematical Reviews* (2000) for B.P. Rynne "The Well-posedness of the Electric Field Integral Equation for Transient Scattering from a Perfectly Conducting Body," *Mathematical Methods in the Applied Sciences* **22** (1999) 619–631.
8. Signed review published as MR99j:76116 by *Mathematical Reviews* (1999) for Y. Xu "Some Inverse Problems in Stratified Media," in H. Florian *et al.* eds., *Generalized Analytic Functions* (Kluwer, Dordrecht, The Netherlands, 1998) 255–264, and also *Int. Soc. Anal. Appl. Comput.,* **1**.
7. Signed review published as MR99c:78009 by *Mathematical Reviews* (1999) for S. He and V.G. Romanov "Identification of Dipole Sources in a Bounded Domain for Maxwell's Equations," *Wave Motion* **28** (1998) 25–40.
6. Signed review published as MR99b:78003 by *Mathematical Reviews* (1999) for L. Demkowicz and L. Vardapetyan "Modeling of Electromagnetic Absorption/ Scattering Problems Using *hp*-adaptive Finite Elements," *Computer Methods in Applied Mechanics and Engineering* **152** (1998) 103–124.

5. Signed review published as MR98j:78010 by *Mathematical Reviews* (1998) for P. Ciarlet, Jr. and E. Sonnendrücker "A Decomposition of the Electromagnetic Field — Application to the Darwin Model," *Mathematical Models and Methods in Applied Sciences* **7** (1997) 1085–1120.

4. Signed review published as MR98d:78007 by *Mathematical Reviews* (1998) for H.-J. Ruppen, "Multiple Cylindrical TM-modes for a Homogeneous, Self-focusing Dielectric," *Journal of Differential Equations* **134** (1997) 286–319.

3. Signed review published as MR98c:78015 by *Mathematical Reviews* (1998) for V.G. Romanov, J. Gottlieb, S.I. Kabanikhin, and S.V. Martakov, "An Inverse Problem for Special Dispersive Media Arising from Ground Penetrating Radar," *Journal of Inverse and Ill-posed Problems* **5** (1997) 175–192.

2. Signed review published as MR97j:78017 by *Mathematical Reviews* (1997). The review is of A. Figotin and A. Klein, "Localized Classical Waves Created by Defects," *Journal of Statistical Physics* **86** (1997) 165–177.

1. Signed review published as MR96j:78001 by *Mathematical Reviews* (1996). The review is of P.-A. Raviart, E. Sonnendrücker, "Approximate Models for the Maxwell Equations," *Journal of Computational and Applied Mathematics* **63** (1995) 69–81.

ring. The ring is 13 mm high and was suspended 1 mm above the ground plane. Good agreement between experimental and simulated results was realised. The measured back lobes were lower than simulated. This can be attributed to the anechoic chamber's mount supporting the antenna while under test. The 3 dB beam width for H and E co-polarised cuts were 80.46° and 73.58°, respectively.

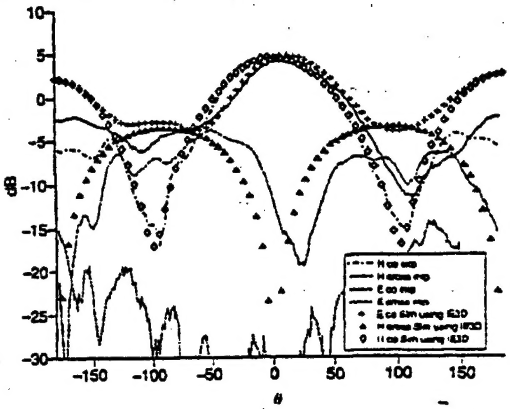

Fig. 4 *Radiation pattern for 20 mm ring on 30 mm ground plane at 2.4 GHz*

Conclusion: A new reduced size antenna has been presented. This new antenna reduces the size of the ground plane to a radius of 0.24λ, at the same time showing large reductions in the back radiation and up to a 5 dB increase in the forward gain when compared to the same antenna without the use of rings. This is better performance than the shorted patch on a ground plane with a radius of 0.63λ or greater. This has allowed a significant decrease in the size of the ground plane and therefore makes it a suitable antenna for PCS devices and other applications.

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Radiated pulses decay exponentially in materials in the far fields of antennas

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There has been recent interest in using short-pulse radar to detect targets in lossy clutter. The analysis presented here shows that the energy and peak-power densities of pulses decay exponentially with depth in homogeneous, lossy, dispersive materials, provided the frequency bands of the pulses are separated from DC. Many numerical examples verify the analytical results.

Introduction: Pulse decay in Lorentz and Debye dispersion models has been studied [1–4] since 1914. Such pulses are said to decay algebraically with depth, typically as $x^{-1/3}$, $x^{-1/2}$, or $x^{-2/3}$ as $x \to \infty$ in these lossy models. This is slower than the exponential decay of single-frequency signals. Several groups recognised this and recently began investigating whether algebraically decaying pulses could usefully penetrate trees or other far-field, lossy clutter in front of targets. As explained in this Letter, it is concluded that the answer is, unfortunately, no.

Algebraic decay is often claimed for pulses with DC or near-DC content. Endnote 30 of [4], e.g. states that a pulse $f(t)$ decays as $x^{-1/3}$ in all undamped Lorentz models if f has DC content; but that the decay is $x^{-2/3}$ if the Fourier transform satisfies $\tilde{f} = 0 \neq d\tilde{f}/d\omega$ at $\omega = 0$, signifying near-DC content. Gauss-modulated cosines ($x^{-1/3}$) and sines ($x^{-2/3}$) are examples. Such pulses propagate well in co-axial cables; however, it is widely known that highly conducting ($0 < \omega\epsilon_0 \ll \sigma < \infty$) dipoles and loops have free-space radiation efficiencies that vanish as $\omega^{3/2}$ and $\omega^{7/2}$, respectively, as $\omega \to 0$. To model radar penetration of far-field clutter, it is therefore assumed here that the pulse spectrum is separated from DC.

Analysis: Let f be any real-valued, band-limited, incident electric-field pulse. Then f propagates in any 1D dispersive half-space $x \geq 0$ as

$$E(x, t) = \int_{-\omega_{max}}^{\omega_{max}} e^{i[k(\omega)x + \omega t]} \tilde{f}(\omega) d\omega \qquad (1)$$

with $k = k_r + ik_i = \omega[\epsilon(\omega)]^{1/2}/c$. Here $\tilde{f}(\omega) = 0$ except where ω satisfies $0 < \omega_{min} \leq |\omega| \leq \omega_{max} \leq \infty$. (A relevant fine point is considered below.) The material is lossy: $k_i > 0$ in $[\omega_{min}, \omega_{max}]$, with extreme values k_i^{min} and k_i^{max}. Standard analysis and (1) yield

$$|E(x, t)| \leq \exp(-k_i^{min} x) \int_{-\infty}^{\infty} |\tilde{f}| d\omega$$

i.e. $|E|$ decays at least as fast as $\exp(-k_i^{min} x)$. The Parseval equation similarly yields

$$\int_{-\infty}^{\infty} |E|^2 dt = \int_{-\infty}^{\infty} |\exp(ikx)\tilde{f}|^2 d\omega \leq \exp(-2k_i^{min} x) \int_{-\infty}^{\infty} |f|^2 dt$$

Thus, energy densities and peak $|E|$ values decay exponentially in all lossy, dispersive materials.

For example, every finite-band pulse separated from DC will decay exponentially in every Debye and damped-Lorentz model. Similar behaviour occurs in the loss bands $b^2 < \omega^2 < a^2 + b^2$ of undamped-Lorentz models $\epsilon = 1 + a^2/(b^2 - \omega^2)$. These examples are unlike the algebraic decay predicted [1–4] for near-DC-content pulses in the same Debye and Lorentz models.

Regarding fine points, the mathematics of entire functions shows that no finite-bandwidth pulse is precisely 0 over any time interval. Yet the results above do hold if $\omega_{max} < \infty$. Finite bands are also practical, and their never-precisely-0 consequences suggest ordinary noise. Indeed, the numerics here will omit features 55 dB below peak power. 'Power' and 'energy' hereafter refer implicitly to densities.

Numerics: The example incident pulses have $\tilde{f}(\omega) = 0$, except for $\omega_{min} \leq |\omega| \leq \omega_{max}$ where $\tilde{f} = \exp\{1 + \omega_b^2/[(|\omega| - \omega_c)^2 - \omega_b^2]\}$ with $\omega_{min} = \omega_c - \omega_b$ and $\omega_{max} = \omega_c + \omega_b$. The computed inverse transform of $\tilde{f}$ is truncated below −55 dB. Conveniently for numerics, this f is briefer than many other pulses with similar spectra. Fig. 1 shows the f used in Fig. 2 for a Debye model. Every f used here for a Lorentz model is in the spectrum of that model's so-called Brillouin precursors, which are often said to decay algebraically.

The Debye model $\epsilon_D = 1 + 58/(1 - i\omega \times 9.4\text{ ps})$ approximates water. Let $u = 1$ rad/s. The damped-Lorentz model $\epsilon_L = 1 + 39 \times 10^{22}u^2/(16 \times 10^{22}u^2 - \omega^2 - 5i\omega \times 10^{10}u)$ is used so ϵ_D and ϵ_L have comparable $k_i(\omega)$ curves in Fig. 3. Indeed, Fig. 3 shows that the bands 13–18 GHz in ϵ_D and 40–50 GHz in ϵ_L have the same k_i^{min} and k_i^{max}. Propagating the corresponding pulses in ϵ_D and ϵ_L yields normalised energies $\mathcal{E}(x) = [\int |E(x, t)|^2 dt]/[\int |f|^2 dt]$ in ϵ_D and ϵ_L that overlap within a line width in the centre of Fig. 2. Normalised peak powers $\mathcal{P}^2(x) = \max_t |E(x, t)|^2/\max_t |f|^2$ in ϵ_D and ϵ_L also overlap $\mathcal{E}$ below $\exp(-2k_i^{min} x)$ in Fig. 2, verifying the analytical results.

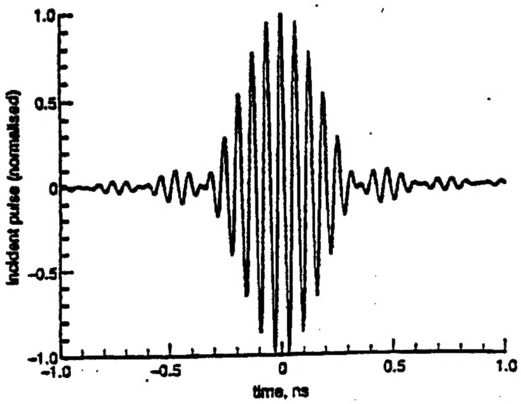

Fig. 1 *Near-peak values of an incident f(t) with parameters $\omega_{min}/2\pi$ = 13.48 GHz and $\omega_{max}/2\pi$ = 18.01 GHz*

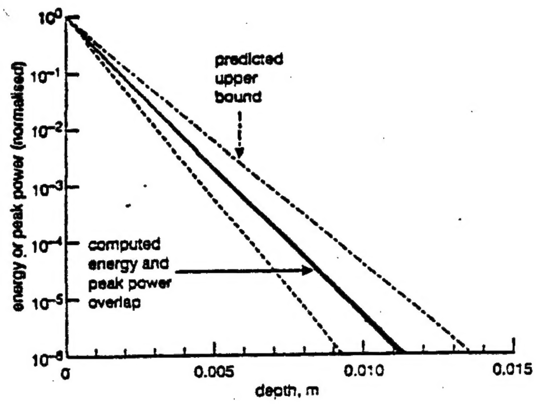

Fig. 2 *Normalised energies and peak powers of a pulse in ϵ_D and another in ϵ_L overlap (centre), are $\le \exp(-2k_i^{min}x)$ (top), and are $\ge \exp(-2k_i^{max}x)$ (bottom)*

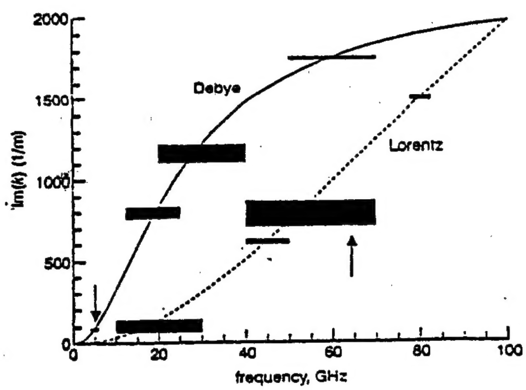

Fig. 3 *Curves $k_i(\omega)$ for models ϵ_D and ϵ_L with exponential decay of peaks verified by eight examples described in text regarding rectangles*

Going beyond the analysis, a more sensitive measure of exponential decay is $-\mathcal{E}'/\mathcal{E} = -d[\ln \mathcal{E}(x)]/dx$. This is -1 times the x-dependent slope (in dB/cm) of $\mathcal{E}(x)$ when graphed in dB. Thus, $-\mathcal{E}'/\mathcal{E}$ is the local exponential decay rate of $\mathcal{E}$. For the pulses of Figs. 1 and 2, the computed $-\mathcal{E}'/\mathcal{E}$ varies from 1185–1257/m (51–55 dB/cm) for the first 60 dB of energy attenuation. This is in the range $[2k_i^{min}, 2k_i^{max}]$ of 1016–1485/m (44–64 dB/cm) suggested but not yet predicted by theory.

Fig. 3 shows ranges of local exponential decay rates $-\mathcal{P}'/\mathcal{P}$ of peaks $\mathcal{P}$ for the first 60 dB of peak-power attenuation. The large, marked rectangle, e.g. signifies that the incident f with spectrum 40–70 GHz has a peak that decays in the Lorentz model ϵ_L at an exponential rate that varies from 709–868/m. The dashed curve shows this is within the range $[k_i^{min}, k_i^{max}]$ of 508–1247/m for ϵ_L. This example and the seven others in Fig. 3 verify the $\le \exp(-k_i^{min}x)$ prediction for peaks. The spectrum for the small, marked rectangle is 4–6 GHz. Linearity and Fig. 3 thereby yield two four-parameter families of examples of peaks that decay faster than $\exp(-k_i^{min}x)$, with bandwidths over three and four octaves.

Conclusions: In this Letter a practical model of pulses in far-field, lossy materials is used to show that exponential decay is typical. This is verified by many numerical examples, with bandwidths up to four octaves. Two other numerical observations have not yet been explained by analysis. First, analysis has not explained why $\exp(-2k_i^{max}x)$ is a bound in Fig. 2. Secondly, the local exponential decay rates of energies and peaks are within bounds suggested, but not yet predicted, by analysis. The analytical results in this Letter, however, are all numerically verified.

A consequence of recent, practical interest is that algebraic decay no longer seems to be a useful design principle for radar penetration of far-field, lossy clutter. Although algebraic decay might be recovered from exponential decay in a mathematical limit $\omega_{min} \to 0$, this would not escape the real difficulties of low radiation efficiency and low resolution posed by near-DC signals.

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High voltage pulse generation

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A new topology of a high voltage source with a variable output pulse pattern and featuring an independent adjustment of the magnitude, repetition frequency and pulse duration is presented. The power stage of the source consists of eight individual unipolar sources that can be arbitrarily connected in series to obtain the desired output voltage pulse of several amps and with extremely high du/dt.

Introduction: Owing to the tremendous increase of applications in oncology, genetics and cell biology, high voltage pulse sources capable of delivering AC or DC currents of several amps are the subject of intensive investigations. Several authors have reported that the application of short high voltage pulses transiently increases the permeability of the cell membrane [1–3]. The so-called electroporation, or electropermeabilisation, has become an effective tool for the internalisation of various molecules, especially anti-cancer drugs and gene material, into the biological cells. The efficiency of such

Derivations of Inequalities for Peaks and Energy Densities

This page adds detail to the derivations in my paper in the July 4, 2002 issue of *Electronics Letters.* In these derivations, Fourier-transform factors of 2π or $\sqrt{2\pi}$ are omitted because they don't affect decay rates.

Each derivation begins with the paper's paragraph that contains equation (1). That paragraph also states assumptions and notations used implicitly by the derivations. It is assumed, for example, that $x \geq 0$.

Peaks

• First, $|E| = |\int \mathrm{e}^{\mathrm{i}(kx+\omega t)}\tilde{f}\mathrm{d}\omega|$ follows immediately from (1).

• Second, $|E| \leq \int |\mathrm{e}^{\mathrm{i}(kx+\omega t)}\tilde{f}|\mathrm{d}\omega$ by a standard inequality for integrals.

• Third, $|E| \leq \int \mathrm{e}^{-k_{\mathrm{i}}x}|\tilde{f}|\mathrm{d}\omega$ because $|\mathrm{e}^{\mathrm{i}\theta}| = 1$ whenever θ is a real number.

• Fourth, $|E| \leq \int \mathrm{e}^{-k_{\mathrm{i}}^{\mathrm{min}}x}|\tilde{f}|\mathrm{d}\omega$ because $\mathrm{e}^{-k_{\mathrm{i}}x} \leq \mathrm{e}^{-k_{\mathrm{i}}^{\mathrm{min}}x}$.

• Finally, $|E| \leq \mathrm{e}^{-k_{\mathrm{i}}^{\mathrm{min}}x}\int |\tilde{f}|\mathrm{d}\omega$ because $\mathrm{e}^{-k_{\mathrm{i}}^{\mathrm{min}}x}$ is independent of ω and may thus be moved outside the integral.

Energy Densities

• First, $\int |E|^2\mathrm{d}t \doteq \int |\mathrm{e}^{\mathrm{i}kx}\tilde{f}|^2\mathrm{d}\omega$ by direct application of the Parseval equation to (1).

• Second, $\int |E|^2\mathrm{d}t = \int \mathrm{e}^{-2k_{\mathrm{i}}x}|\tilde{f}|^2\mathrm{d}\omega$ because $|\mathrm{e}^{\mathrm{i}\theta}| = 1$ whenever θ is a real number. Algebra is also used here.

• Third, $\int |E|^2\mathrm{d}t \leq \int \mathrm{e}^{-2k_{\mathrm{i}}^{\mathrm{min}}x}|\tilde{f}|^2\mathrm{d}\omega$ because $\mathrm{e}^{-2k_{\mathrm{i}}x} \leq \mathrm{e}^{-2k_{\mathrm{i}}^{\mathrm{min}}x}$.

• Fourth, $\int |E|^2\mathrm{d}t \leq \mathrm{e}^{-2k_{\mathrm{i}}^{\mathrm{min}}x}\int |\tilde{f}|^2\mathrm{d}\omega$ because $\mathrm{e}^{-2k_{\mathrm{i}}^{\mathrm{min}}x}$ is independent of ω and may thus be moved outside the integral.

• Finally, $\int |E|^2\mathrm{d}t \leq \mathrm{e}^{-2k_{\mathrm{i}}^{\mathrm{min}}x}\int |f|^2\mathrm{d}t$ because (1) at $x = 0$ yields $f = \int \mathrm{e}^{i\omega t}\tilde{f}\mathrm{d}\omega$ and then the Parseval equation implies $\int |\tilde{f}|^2\mathrm{d}\omega = \int |f|^2\mathrm{d}t$.

The Parseval equation is in many references on Fourier transforms and it is in some math tables, *e.g.*, M.R. Spiegel, *Mathematical Handbook of Formulas and Tables* (Schaum's Outline Series, McGraw-Hill). In the 1991 edition, the Parseval equation is Eq. 33.11 on p. 175.

END